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OPTIMAL STRATEGIES AND HUMAN BEHAVIOR IN FUNGUS-EATER GAME 4

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Jun-ichi Nakahara
Masanao Toda

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FOREWORD

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ABSTRACT

Fungus-eater games yield a class of sequential decision tasks which involve both means objects and end objects in a not-too-unrealistic fashion. Study of human strategies and of the optimal strategies in these games may improve our understanding of complex dynamic decision tasks.

The optimal strategy for the fourth fungus-eater game, which depends upon the level of fungus storage, is derived and the behavior of human playing this game is reported.

PUBLICATION REVIEW AND APPROVAL

This Technical Documentary Report has been reviewed and is approved.

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HERBERT RUBENSTEIN
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Decision Sciences Laboratory

Roy Morgan
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OPTIMAL STRATEGIES AND HUMAN BEHAVIOR IN FUNGUS-EATER GAME 4

Jun-ichi Nakahara and Masanao Toda

In this paper we derive the optimal strategies for the finite, non-coexistent V-span 1 game which we call game 4, G4. The reader is assumed to be familiar with the three preceding papers of this series (Toda, 1962; 1963a; and 1963b) but just for refreshing the reader's memory a brief summary of the first issues cited above, in particular, the part relevant to the present article, will be presented in the first section of this paper.

1. The structure of discrete F-E games and their optimal strategies

A F-E game is said to be discrete if the F-E is allowed to move only along a certain branch structure. Each branching point of the branch structure is said to be a choice point for the F-E. Retracing of the same branch is forbidden.

If there exists always two new branches at each choice point, the game is said to be binary. When the two new branches always converge to the same choice point, the branch structure is said to be a chain. There are several kinds of branch structure[1], and one theorem concerning the influence of branch structure.

THEOREM 1: <u>If the environment is binary and homogeneous, and the size of V-span is one, the optimal decision function for a well-informed F-E is independent of branch structure of the environment.</u>

---

[1] See Toda (1963a).

Proof of this theorem may be found in Toda (1963a) and the notion of homogeneous, well-informed, and V-span will be explained later in this section.

G4 is a binary, homogeneous, V-span 1 game.

The part of the branch starting from the choice point and ending at the next choice point, but not including the choice points themselves, is called a path. The whole set of paths starting from the same choice point is said to form a unit environment. In a binary game, every unit environment consists of two paths.

The two kinds of substances which can exist in a unit environment are F(fungus) and U(uranium). They do not exist on choice points but on paths.

A F-E game is said to be singular if not more than one U and not more than one F can exist in any single unit environment provided by the game. If to a singular game is added one more constraint such that not more than one substance can exist on the same path, the game is said to be exclusive. G4 is a singular and exclusive game.

If the environment is singular and exclusive, there can be only four types of entries in each unit environment, (F U), (F O), (O U), and (O O), where (F U) means that one fungus exists on one of the paths constituting the unit environment and one uranium exists on the other path. (F O) means that fungus exists on one of the paths, and nothing on the other. And so on.

If the probability distribution over the alternative types of unit environment encountered at each choice point is independent of the choice point, the game is said to be homogeneous. G4 is homogeneous.

The state of F-E (a player of a F-E game) is characterized by three variables, i.e., the F-storage, the U-storage, and the L-storage.

At the beginning of a play, F-E is given certain initial amount of these three variables which may be finite or infinite. The F-storage regularly decreases by one unit as F-E moves from one choice point to the next. Whenever one F is picked up by F-E on the way, F-storage increases by $\underline{a}$ units. Therefore, the net gain of F-storage through the unit locomotion is $\underline{a-1}$ units. When F-storage reaches zero, the play is finished. The U-storage increases by one unit whenever F-E takes one U. The L-storage, like F-storage, decreases by one unit as F-E moves from one choice point to the next. But, there is nothing, like fungus for F-storage, that increases L-storage. When L-storage reaches zero, the play of the game is finished.

According to whether the initial L-storage is finite or infinite, the F-E game is said to be finite game or infinite game. G4 is a finite game.

In G4 we also assume that the initial F-storage is finite, and that the initial U-storage is zero.

When F-E is provided all necessary information concerning the structure of the game including the probability distributions of fungus and uranium on the unit environment, the F-E is said to be well-informed. The F-E in G4 is well-informed. Besides knowledge of the probability distribution, F-E can see, in general, the contents of unit environments near his choice point. If he sees starting from the unit environment belonging to his present choice point (the immediate unit environment) up to those belonging to the n-th possible choice points, the F-E is said to be of V-span $\underline{n}$. The F-E in G4 is of V-span 1.

The pay-off to F-E is proportional to the U-storage at the time when play is finished.

The well-informed F-E has knowledge about the defining characteristics of the game, himself, and the environment, in particular, the probabilities governing the distributions of F and U in the unit environment. We call this set of knowledge the permanent decision context, P. If F-E has vision, he has at each choice point the information about the content in terms of F and U of the environment covered by his V-span. We call this set of knowledge of the external decision context, E.

Furthermore, with the progress of the game, the internal state of the F-E defined by the variables like his F-storage, L-storage, and U-storage will change. We will call this set of knowledge about the F-E himself the internal decision context, I.

A strategy, or we may call it a decision function, is a set of rules that dictates the F-E's decision at each choice point, and since each choice point is virtually defined by the three decision contexts, P, E, and I, a strategy is a function of these three decision contexts, i.e.,

$$D = D(P, E, I) \quad .$$

The optimal strategy, or the optimal decision function of the given game, is the strategy that maximizes the expected future U return. If F-E is well-informed, the expectation is the mathematical expectation in the ordinary sense.

After a game is specified, the permanent decision context will no longer be variables. It comes into the game as a set of parameters.

Decision contexts, I and E, vary at each choice point. The value of the decision function, D, is a decision. That is, it will take a value F or U according to the values of the three decision contexts, and it is unique when I and E are fixed. If a decision function maximizes the future U return at each choice point, then it is the optimal decision function.

We define the expected future U return function, V, as a functional of the decision function and the three decision contexts when the F-E is well-informed.

$$V = V \ \{D(P,\ I,\ E)\ ;\ P,\ I,\ E\ \} \quad .$$

V will attain its maximum when the decision function is the optimal one. Thus, the problem is to find the optimal decision function which maximizes the future U return, i.e.,

$$V^* = \underset{\{D\}}{\mathrm{Max}}\ V(D;\ P,\ I,\ E) \tag{1}$$

2. IC diagram and some remarks concerning G4

Let us briefly talk about the IC (Internal Context) diagram which was introduced to the F-E game in G3 (Toda, 1963a). Our problem may most conveniently be visualized by using this diagram.

This diagram shows the internal state of the F-E by a point located at one of the intersections of the grid shown in Fig. 1.

As the F-E travels one step in the environment, the point moves either upward or to the right one unit, depending upon whether he takes F or not on the path to the next choice point.

If the point reaches the line on the lower right of the diagram labeled "starvation absorption barrier," the point is absorbed there indicating that the F-E dies there by starvation.

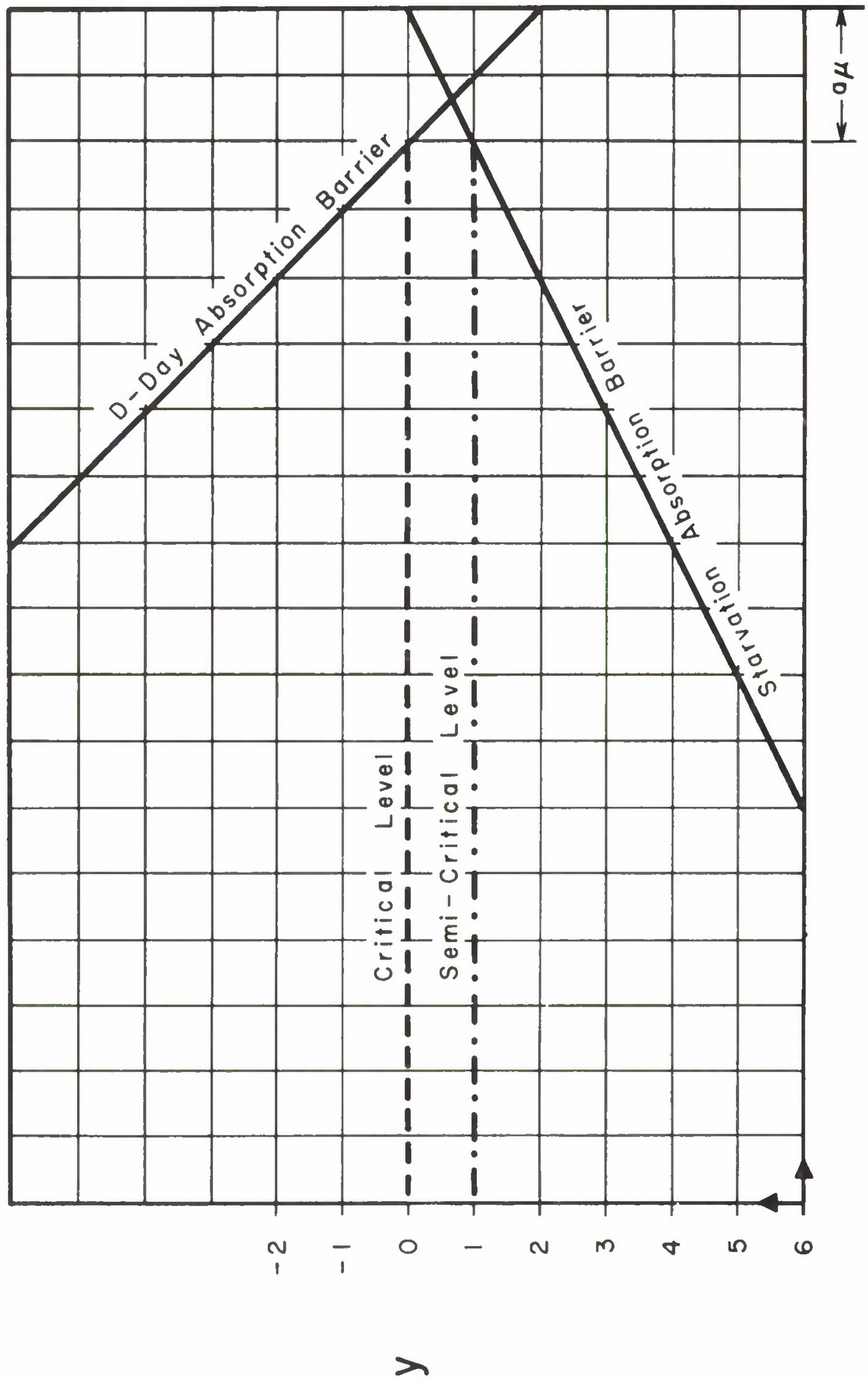

D-Day Absorption Barrier
Starvation Absorption Barrier
Critical Level
Semi-Critical Level
$a\mu$
-2
-1
0
1
2
3
4
5
6
y

FIG. 1

The distance, $\underline{x}$, to the starvation absorption barrier along a line parallel to the abscissa represents F-E's F-storage, and thus $\underline{x}$ is equal to zero on the starvation line. Suppose a F-E starting with $\underline{x}$ F-storage never takes fungus at all. Then the point of his internal state keeps moving to the right one unit on each trial, and reaches the starvation line. Certainly he must die after $\underline{x}$ trials if he never takes fungus during the trip. So, any point located $\underline{x}$ units to the left of the starvation absorption line represents the internal context corresponding to the F-storage containing fungi sufficient for the F-E to negotiate $\underline{x}$ trials. It is easily seen in the diagram that taking a fungus increases F-storage $\underline{a-1}$ units.

Another absorption barrier in the figure is the "Dooms Day absorption barrier." Any F-E with finite L-storage must die after traveling $\underline{n}$ steps, where $\underline{n}$ is a finite integer. He may die earlier by starvation, but he must die by $\underline{n}$ steps regardless of the decisions he made. So any point in this diagram representing the internal context $\underline{n}$, namely L-storage, must locate at the distance of $\underline{n}$ units away (independently from the direction of movement) from this barrier. From this consideration the inclination of this barrier is automatically determined as shown in Fig. 1.

The broken line in the middle of the diagram at $y = 0$ is called the critical level, and the broken line one unit below the critical line is called the semi-critical level. Let us define any point locating $\underline{y}$ units below the critical level as representing the new relative internal context $\underline{y}$. We call $\underline{y}$ the F-need, because the F-E is required to take at least $\underline{y}$ fungi to meet the D-day barrier. We will talk about $\underline{y}$ more explicitly in a later section.

Thus, each different internal context will explicitly be represented by a point in the IC diagram.

We define G4 as a finite, non-coexistent V-span 1 game, and the environment of this game is characterized as singular, and thus binary, exclusive, and homogeneous. Therefore, there are four types of unit environments, (F U), (F O), (U O), and (O O), to which we assign probabilities $\underline{fu}$, $\underline{f(1-u)}$, $\underline{(1-f)u}$, and $\underline{(1-f)(1-u)}$, respectively, and which also exhaust the external decision contexts. Here, as indicated by the probability assignment, we assume independence between F distribution and U distribution over unit environment. The probability that F is found in a unit environment is $\underline{f}$, and the probability for U in a unit environment is $\underline{u}$. These two probabilities belong to the permanent decision context. The other parameter of the permanent decision context is $\underline{a}$ which represents the increase in F-storage due to taking one fungus.

The parameters $\underline{u}$, $\underline{f}$, and $\underline{a}$ constitute the permanent decision context. We have no other element of permanent decision context, so we may express the decision function as

$$D(z, \mathbf{I};\ f,\ u,\ a)$$

where $\underline{z}$ is a random variable and takes one of four values, (F U), (F O), (O U), or (O O) with probability $\underline{fu}$, $\underline{f(1-u)}$, $\underline{(1-f)u}$, or $\underline{(1-f)(1-u)}$ respectively, and $\underline{I}$ represents the internal decision context. When the value of $\underline{z}$ is either (F O), (O U), or (O O), the optimal decision is obvious, so that it is sufficient to solve the decision function for $\underline{z}$ = (F U).

Any trial in which the external decision context (F U) is given to F-E is a (non-trivial) decision trial.

In G4 there is no internal context which is relevant to the decision function other than x, F-E's F-storage in tuts[1] unit, and y, F-E's F-need to see the "Dooms Day absorption barrier."

Let us have a short descritpion about y, because we use y instead of n, F-E's L-storage, for describing the internal decision contexts.

Suppose a F-E starts this F-E game with finite L-storage n and finite F-storage x. Assume $n > x$. Then, to live until Dooms-Day, the minimum number of F he must take is $(n - x)/a$. If this quotient comes out to be an integer, he is able to see that D-day without having any left-over F-storage. In this case we say that the D-day is in phase, and if not, it is out of phase. If the D-day is out of phase, the minimum number of F he needs to see the D-day is the smallest integer greater than the above quotient, and he will have some amount of left-over F-storage.

Now y is defined as

$$y - \mu = \frac{n - x}{a}$$

where y is an integer which shows the minimum number of F necessary to see the D-day as we described before, and μ is the phase. It takes on values between 0 and 1, $0 \leq \mu \leq 1$.

The phase is not so important a factor in determining the over-all decision strategy, but still we cannot ignore it. Sometimes it determines the relative value of the last F to see the D-day. If μ is very close to one, almost all of the contents of the last F will be left-over.

[1] See Toda (1963a)

The value of $\underline{\mu}$ is determined completely by F-E's initial F-storage and initial L-storage, and remains unchanged throughout the rest of the game.

What changes with age is $\underline{y}$. The pair of variables (x, y) may as well describe the internal decision context as (x, n) and we will use the former in what follows.

3. Optimal decision function

As we have already shown that the only external decision context relevant to the optimal decision function is z = (F U). So our problem is to assign the optimal decision to each point of the IC diagram, i.e., the decision that is effective when z = (F U) is given at that point.

Now what are the alternative decisions? There are decision F, decision U, and all kinds of mixed decisions. Fortunately, however, according to the Theorem 7 given in Toda (1963a) we need not worry about mixed decisions in finite games.

Thus, our problem is to find the optimal decision at each point on the IC diagram, decision "F" or decision "U", which maximizes the expected future U return function V.

Let us give a much more explicit expression to the equation (1). We can rewrite (1) as

$$V^*(x\ ,\ y) = \text{Max}\ \left\{ V(D_F;\ x,y)\ \ ,\ \ V(D_U;\ x,y) \right\} \tag{2}$$

where D_F (or D_U) is such a strategy that is identical to D^*, the optimal decision function, except at I = (x , y) where it dictates to take F (or U), if possible, whether it is optimal or not.

If actually F-E takes F, his internal context at the next choice point is $I = (x+a-1, y-1)$ by the definition. If $z = (O\ U)$ is given, he takes U and proceeds to the next choice point and his internal decision context will be $I = (x - 1, y)$. If $z = (O\ O)$ is given, he just proceeds to $I = (x - 1, y)$. Then, we have the explicit expression for $V(D_F; x, y)$ as

$$\begin{aligned} V(D_F; x, y) &= fuV^*(x + a - 1, y - 1) \\ &\quad + f(1 - u)V^*(x + a - 1, y - 1) \\ &\quad + (1 - f)u\ \{V^*(x - 1, y) + 1\} \\ &\quad + (1 - f)(1 - u)V^*(x - 1, y) \\ &= fV^*(x + a - 1, y - 1) \\ &\quad + (1 - f)V^*(x - 1, y) + (1 - f)u \quad . \end{aligned}$$

Analogously, for $V(D_U; x, y)$

$$\begin{aligned} V(D_U; x, y) &= fu\ \{V^*(x - 1, y) + 1\} \\ &\quad + (1 - f)u\ \{V^*(x - 1, y) + 1\} \\ &\quad + f(1 - u)V^*(x + A - 1, y - 1) \\ &\quad + (1 - f)\ (1 - u)V^*(x - 1, y) \\ &= f(1 - u)V^*(x + a - 1, y - 1) \\ &\quad + (1 - f + fu)V^*(x - 1, y) + u \quad . \end{aligned}$$

By taking the difference between these two expected U gain functions, we can determine which one gives the greater expectation.

$$\begin{aligned} &V(D_F; x, y) - V(D_U; x, y) \\ &\qquad = fu\ \{V^*(x + a - 1, y - 1) - V^*(x - 1, y) - 1\} \ . \end{aligned}$$

So by defining a delta function $\delta(x, y)$ as follows

$$\delta(x, y) = V^*(x + a - 1, y - 1) - V^*(x - 1, y) - 1 \quad , \tag{3}$$

the value of optimal decision function for a given internal decision context is definitely specified as follows

$$(4) \qquad D^*(z = (F\ U);\ x,\ y) \begin{aligned} &= F \quad \text{if} \quad \delta(x\ ,\ y) > 0 \\ &= O \quad \text{if} \quad \delta(x\ ,\ y) = 0 \\ &= U \quad \text{if} \quad \delta(x\ ,\ y) < 0\ , \end{aligned}$$

where $D^* = 0$ means indifference between F and U.

4. Analytical solution

Now, just for the sake of simplicity, let us assume that the D-day is in phase. Then $\underline{y = 0}$ is the only critical level. [1]

By definition of $\underline{y}$, the F-E whose value of $\underline{y}$ is zero can survive up to D- day with probability 1. So giving up U for F when $z = (F\ U)$ is obviously sub-optimal. Theorem 8 given in Toda's paper [2] states this explicitly as

THEOREM 8: $\underline{D^*(z = (F\ U);\ x,\ y) = U \ \text{ if } \ y \le 0 \text{ and } n > 0}$

The expected U gain function for the critical level is directly derived from this Theorem as

$$(5) \qquad V^*(\ x,\ y = 0\ ) = ux$$

Note that we are assuming $\mu = 0$.

Now we have Theorem 8 and the equation (5) as the boundary conditions, and we proceed to solve the optimal decision function for $\underline{y = 1}$.

---

1 This will not be too much a simplification. It can be easily shown that the same procedure that we are going to use to solve this problem is applicable to the case of out of phase D-day.

2 Toda (1963a)

Now, some really interesting features appear in G4. For example, the optimal decision not only depends on $\underline{y}$ but also on $\underline{x}$. This has never been the case through G1 to G3.[1]

By putting y = 1 in (3) we have

$$\delta\,(x,\ y = 1) = V^*(x + a - 1,\ y = 0) - V^*(x - 1,\ y = 1) - 1 \quad .$$

What we want to solve for is that $\underline{x}$ which gives the neutral optimal decision, namely the value of $\underline{x}$ with which the optimal decision is 0, $\delta\,(x,\ y) = 0$ [2]. Let us denote this value of $\underline{x}$ as $\underline{x}_1$. Then, if $\underline{x}$ is greater than $\underline{x}_1$ the optimal decision should be U, and if $\underline{x}$ is smaller than $\underline{x}_1$, the optimal decision should be F [3]. Thus, this $\underline{x}_1$ is the critical decision-shifting point on the line $\underline{y = 1}$. This $\underline{x}_1$ will divide the line $\underline{y = 1}$ into two parts characterized by different optimal decisions.

By obtaining the decision-shifting point $\underline{x}_2$ for y = 2, $\underline{x}_3$ for y = 3, and so on, the whole IC diagram will be divided into two regions, namely the U decision region and the Fdecision region.

Thus, obtaining the decision-shifting point for each value of $\underline{y}$ is all we need for the optimal strategy in G4. In other words, our problem is to solve $\underline{x}$ satisfying the following equation for each value of $\underline{y}$, y > 0:

$$D^*\ \{\, z = (F\ U);\ x,\ y \,\} \quad = 0 \quad ,$$

or

$$\delta\ (\,x,\ y) \ = \ 0 \quad . \tag{6}$$

---

1 See the former report (Toda, 1963a)

2 Though $\underline{x}$ is an integer in the discrete game like G4, here we regard $\underline{x}$ as a continuous variable for convenience.

3 Actually we not only assume the uniqueness of the solution $\delta\,(\,x,\ y) = 0$ with respect to $\underline{x}$, but also we assume the optimal decision F on each $\underline{y}$ line between the decision-shifting point and the starvation absorption line, and the optimal decision U on other parts of $\underline{y}$ line.

Then, $\underline{x}_1$ is the solution of the equation (7),

$$(7) \qquad \delta(x, y=1) = V^*(x+a-1, y=0) - V^*(x-1, y+1) - 1 = 0 \quad .$$

From (5) the first term of (7) is given as

$$(8) \qquad V^*(x+a-1, y=0) = u(x+a-1) \quad .$$

If $\underline{x}$ is neutral decision point, $\underline{x-1}$ surely falls into the F decision region. Therefore, $V^*(x-1, y=1)$ is the expected U gain function in the F decision region, which means that the optimal F-E must take F whenever he comes to a fungus. So we have

$$(9) \qquad V^*(x-1, y=1) = u(x-1)(1-f)^{x-1} + u(x+a-2)\left\{1-(1-f)^{x-1}\right\}$$

where the first term shows the expected U gain under the condition that the F-E dies on the line $\underline{y=1}$, and the second term shows the expected U gain under the condition that the F-E takes one fungus and goes up to $\underline{y=0}$ line. By substituting (8) and (9) into (7), we have

$$\delta(x, y=1) = u(x+a-2)(1-f)^{x-1} - u(x-1)(1-f)^{x-1} + u - 1 = 0 \quad .$$

Therefore,

$$(10) \qquad (1-f)^{x-1} = \frac{1-u}{u(a-1)} \quad .$$

For convenience let us regard $\underline{x}$ as if it is continuous. Then we have

$$(11) \qquad (x-1)\log(1-f) = \log(1-u) - \log u - \log(a-1) \quad .$$

Therefore

$$(12) \qquad x = \frac{\log(1 - u) - \log u - \log(a - 1)}{\log(1 - f)} + 1 \quad .$$

Thus, $\underline{x}_1$ is obtained.

What we shall do next will be to obtain the decision-shifting point $\underline{x}_2$ for $\underline{y = 2}$ and higher values. But as one will soon see, to solve the decision-shifting points for higher values of $\underline{y}$ is not so easy as it was for $\underline{y = 1}$. So we will just briefly portray an outline of the procedure for obtaining the analytical solution of $\underline{x}_2$ below, and will proceed to the next section where we shall discuss the numerical method for obtaining the decision-shifting points for higher values of $\underline{y}$.

The delta function for $\underline{y = 2}$ is given as

$$(13) \qquad \delta(x,\ y = 2) = V^*(x + a - 1,\ y = 1) - V^*(x - 1,\ y = 2) - 1 \quad .$$

Let us define the ∂ areas and the β areas in the IC diagram as they are illustrated in Fig. 2.

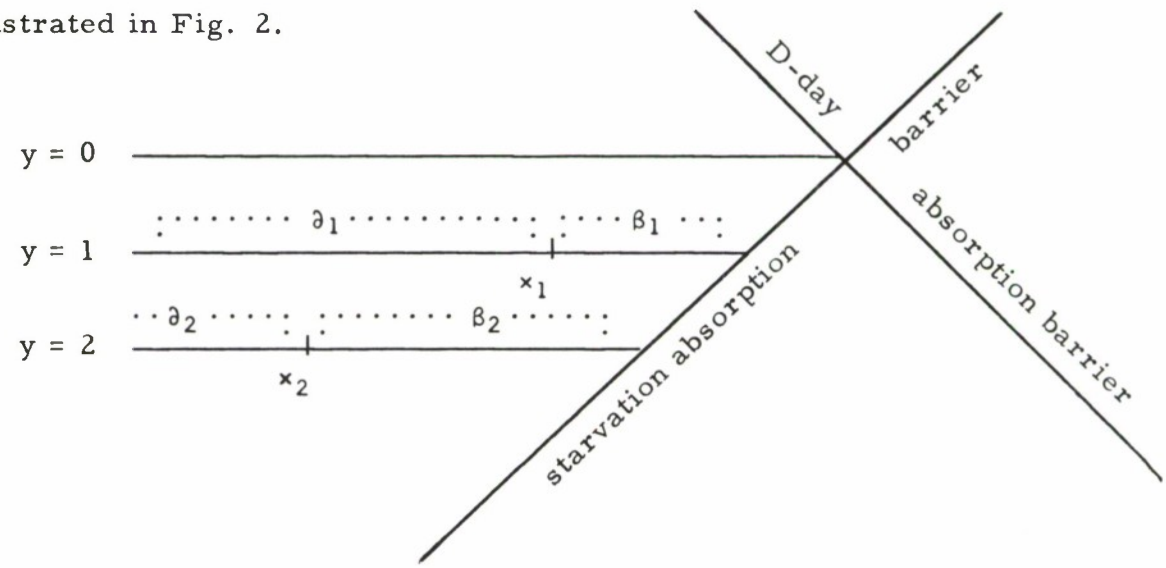


Fig. 2

Usually the solution $\underline{x}_1$ of the equation (12) will not be an integer, but from now on, when we refer to $\underline{x}_1$, we shall mean the integral part of the solution $\underline{x}_1$ of (12). Then $I = (x, y = 1)$ locates in the F decision region and $I = (x + 1, y = 1)$ in the U decision region[1]. Consider the expected U gain function $V^*(x + a - 1, y = 1)$ which is the first term in the right hand side of (13) for $\underline{x} = \underline{x}_2$ for the following three alternative cases:

Case 1F: The F-E at $I = (x + a - 1, y = 1)$ dies on the line $\underline{y = 1}$.

The probability that the Case 1F becomes true is

$$(1 - f)^{x_1} \cdot (1 - f + uf)^{x_2 + a - 1 - x_1}$$

and the expected U gain given Case 1F is

$$(1 - f)^{x_1} \cdot (1 - f + uf)^{x_2 + a - 1 - x_1} \cdot u(x + a - 1)$$

Case 2F: The F-E climbs one level up from the area β_1 to the line $\underline{y = 0.}$

The probability that the Case 2F becomes true is

$$(1 - (1 - f)^{x_1}) \cdot (1 - f + uf)^{x_2 + a - 1 - x_1}$$

and the corresponding expected U gain is

$$\{1 - (1 - f)^{x_1}\} \cdot (1 - f + uf)^{x_2 + a - 1 - x_1} \cdot u(x + 2a - 2) \quad .$$

Case 3F: The F-E climbs one level up from the area ∂_1 to the line $y = 0$.

The probability that the Case 3F becomes true is

$$1 - (1 - f + uf)^{x_2 + a - 1 - x_1}$$

---

[1] When the solution $\underline{x}_1$ itself is an integer, $I = (x \quad, y = 1)$ lies on the border of F and U regions. No harm is done, however, by stipulating that the border itself belongs to the F decision region.

and the corresponding expected U gain is

$$\{1 - (1 - f + uf)^{x_2 + a - 1 - x_1}\}. \; u(x + 2a - 2) \quad .$$

Therefore, $V^*(x + a - 1, \; y = 1)$ is expressed as the sum of expected U gain under these three cases:

$$\begin{aligned} V^*(x + a - 1, \; y = 1) = \; & (1 - f)^{x_1} \cdot (1 - f + uf)^{x_2 + a - 1 - x_1} \cdot u(x + a - 1) \\ & + \{ 1 - (1 - f)^{x_1}\} \cdot (1 - f + uf)^{x_2 + a - 1 - x_1} \cdot u(x - 2a - 2) \\ & + \{ 1 - (1 - f + uf)^{x_2 + a - 1 - x_1}\} \cdot u(x + 2a - 2) \\ = \; & u(x + 2a - 2) \\ & + (1 - f)^{x_1} \cdot (1 - f + uf)^{x_2 + a - 1 - x_1} \cdot u(1 - a) \quad . \end{aligned}$$

Our next step is to obtain an explicit expression for $V^*(x - 1, \; y = 2)$, the second term of (13), and this will again be done considering the following alternative cases separately. It is clear that if $\underline{x_2}$ is the decision-shifting point, $\underline{x_2 - 1}$ surely falls into the F decision region.

Case 1U: The F-E dies on the line $y = 2$. The probability that the Case 1U becomes true is

$$(1 - f)^{x_2 - 1}$$

and the expected U gain is

$$(1 - f)^{x_2 - 1} \cdot u(x_2 - 1) \quad .$$

Case 2U: The F-E dies on the line $\underline{y = 1}$. This case may further be classified into the following subclasses:

Case 2U ∂ : The F-E goes up to the line $\underline{y = 1}$ from the area ∂_2 and dies on that line. The probability that the Case 2U ∂ will happen is

$$f(1 - f)^{x_2 + a - 2} \cdot (x_2 - a + 1)$$

and the expected U gain is

$$f(1 - f)^{x_2 + a - 2} \cdot (x_2 - a + 1) \cdot u(x_2 + a - 2) \quad .$$

Case 2U β : The F-E goes up to the line $y = 1$ from the area β_2 and dies on that line. The probability for the Case 2U β is

$$\frac{1}{u} (1 - f + uf)(1 - f)^{x_1} \{(1 - f + uf)^{x_2 + a - 2 - x_1} - (1 - f)^{x_2 + a - 2 - x_1}\}$$

and the expected U gain is

$$\frac{1}{u} (1 - f + uf)(1 - f)^{x_1} \{(1 - f + uf)^{x_2 + a - 2 - x_1} - (1 - f)^{x_2 + a - 2 - x_1}\}$$
$$\cdot u(x_2 + a - 2) \quad .$$

Case 3U: The F-E goes up to the line $y = 1$, and further goes up to the line $y = 0$. This case must also be subdivided.

Case 3U∂ : The F-E goes up to the line $y = 1$ from the area ∂_2, and further goes up to the line $y = 0$ from the area ∂_1. The probability for the Case 3U ∂ is

$$(1 - f)^{x_2 + a - 2 - x_1} - (1 - f)^{x_2 - 1} - f(1 - f)^{x_2 + a - 2} \cdot (x_1 - a + 1)$$

and the expected U gain is

$$\{(1 - f)^{x_2 + a - 2 - x_1} - (1 - f)^{x_2 - 1} - f(1 - f)^{x_2 + a - 2} \cdot (x_1 - a + 1)\}$$
$$\cdot u(x_2 + 2a - 3) \quad .$$

Case 3Uβ : The F-E goes up to the line $y = 1$ from the area β_2, and further goes up to the line $y = 0$ from the area β_1. The probability for the Case 3U β is

$$\{1 - (1 - f)^{x_2 + a - 2 - x_1}\}$$
$$- \frac{1}{u} (1 - f + uf) \{(1 - f + uf)^{x_2 + a - 2 - x_1} - (1 - f)^{x_2 + a - 2 - x_1}\}$$

and the expected U gain is

$$[\{ 1 - (1 - f)^{x_2 + a - 2 - x_1}\} - \frac{1}{u}(1 - f + uf) \{ (1 - f + uf)^{x_2 + a - 2 - x_1}$$

$$- (1 - f)^{x_2 + a - 2 - x_1} \}] \cdot u(x + 2a - 3) \quad .$$

<u>Case 3U $\beta\partial$:</u> The F-E goes up to the line $y = 1$ from the area β_2, and further goes up to the line $y = 0$ from the area ∂_1. The probability for the Case 3U$\beta\partial$ is

$$\frac{1}{u} (1 - f + uf) \{ 1 - (1 - f)^{x_1}\}\{(1 - f + uf)^{x_2 + a - 2 - x_1}$$

$$- (1 - f)^{x_2 + a - 2 - x_1}\} \quad ,$$

and the expected U gain is

$$\frac{1}{u} (1 - f + uf) \{ 1 - (1 - f)^{x_1} \} \{ (1 - f + uf)^{x_2 + a - 2 - x_1}$$

$$- (1 - f)^{x_2 + a - 2 - x_1}\} \cdot u(x_2 + 2a - 3) \quad .$$

By summing up all these expected U gains we have an explicit expression for $V^*(x_2 - 1, y = 2)$;

(15) $$V^*(x_2 - 1, y = 2) = u(x_2 + 2a - 3) - 2u(a - 1)(1 - f)^{x_2 - 1}$$

$$- (a - 1)(1 - f)^{x_1} (1 - f + uf)^{x_2 + a - 1 - x_1}$$

$$+ (a - 1)(1 - f)^{x_2 + a - 1} \quad .$$

Thus, finally, the delta function for <u>$y = 2$</u> is expressed as

(16) $$\delta(x_2 , y = 2) = V^*(x_2 + a - 1, y = 1) - V^*(x_2 - 1, y = 1) - 1$$

$$= (a - 1)(1 - f)^{x_1} (1 - u)(1 - f + uf)^{x_2 + a - 1 - x_1}$$

$$+ (a - 1)(1 - f)^{x_2 - 1} \{ 2u - (1 - f)^{a} \} - (1 - u) \quad .$$

The solution $\underline{x}_2$ is therefore obtained as the maximum integer which makes $\delta(x, y = 2)$ given in (16) non-negative. The tedious procedure thus portrayed for obtaining $\underline{x}_2$ is certainly enough to discourage us from proceeding any further to seek the solution for $\underline{y = 3}$ and higher values. In the next section, therefore, we turn our attention to numerical methods which will give us the decision-shifting points of higher orders for a given set of parameters (permanent decision context) with the aid of a computer.

5. The recurrent relation of the expected U gain functions

The computational procedure of our numerical method will most conveniently be described in terms of the recurrent relation of the expected U gain functions expressed as follows:

$$(17) \qquad V^*(x, y) = t^*(x, y) \cdot V^*(x + a - 1, y - 1) + \bar{t}^*(x, y) \cdot V^*(x - 1, y) + U^*(x, y)$$

where $t^*(x, y)$ is the probability of taking fungus under the given optimal strategy for the internal decision context $I = (x, y)$, and $\bar{t}^*(x, y)$ is the complement of $t^*(x, y)$. $U^*(x, y)$ is the expected U gain under the same strategy for the unit travel from $I = (x, y)$ to $I = (x - 1, y)$.

When F-E is in the F decision region, he will take fungus whichever of the external decision contexts, $z = (F\ O)$ and $z = (F\ U)$, occurs. Therefore, $t^*(x, y)$ is exactly $\underline{f}$ as long as F-E is in the F decision region, and he proceeds to the next choice point where his internal decision context is expressed as $I = (x + a - 1, y - 1)$ with probability $\underline{f}$. Furthermore, if there happens no external decision context which contains fungus as its entry when F-E's internal decision context is $I = (x, y)$, he just proceeds one step toward the starvation line where his internal decision context is $I = (x - 1, y)$, and he may luckily pick up a piece of uranium with probability $u(1 - f)$, this is the probability of $\underline{z}$ taking the value $z = (O\ U)$. Therefore, $U^*(x, y)$ is equal to $u(1 - f)$ when F-E is in the F decision region.

Analogously when F- E is in the U decision region, he will take fungus only if the external decision context $z = (F\ O)$ is given, and, therefore, $t^*(x, y)$ is equal to $\underline{f(1 - u)}$. Also $U^*(x, y)$ is equal to $\underline{u}$ when F-E is in the U decision region. Now, the foregoing can be abbreviated as follow:

(18) $t^*(x, y) = f$ if $I=(x, y)$ is in the F decision region

$= f(1 - u)$ if $I=(x, y)$ is in the U decision region

(19) $U^*(x, y) = u(1 - f)$ if $I=(x, y)$ is in the F decision region

$= u$ if $I=(x, y)$ is in the U decision region

6. A numerical method for obtaining the optimal solution

By virtue of Theorem 8[1] the optimal decision on the critical level, namely when $y = 0$, is always the decision U. Therefore, the expected U gain for $I = (x, y = 0)$ is a linear function of $\underline{x}$, as given in (5). To restate:

(5) $$V^*(x, y = 0) = ux ,$$

where $\mu = 0$ is assumed.

On each absorption barrier, there is no further U return. This yields the two boundary conditions, namely

(20) $$V^*(n = 0) = 0 ,$$

---

1 Toda op. cit.

(21) $$V^*(x = 0,\ y) = 0 \quad .$$

Since the optimal decisions and the expected U gain function under the optimal strategy for $y = 0$ are already given, let us proceed to the case when $y = 1$.

When $I = (1\ ,\ 1)$, the delta function will be expressed as

$$\delta(1\ ,\ 1) = V^*(a\ ,\ 0) - V^*(0\ ,\ 1) - 1 \quad .$$

By the boundary conditions and the equation (5), the above equation can be re-written as

$$\delta(1\ ,\ 1) = ua - 1 \quad .$$

Therefore, if $ua - 1 \geq 0$[1], the optimal decision is F and $I = (1\ ,\ 1)$ belongs to the F decision region. Thus, by putting $t^*(1\ ,\ 1) = f$, and $U^*(1\ ,\ 1) = u(1 - f)$ we have from (17)

$$V^*(1\ ,\ 1) = f\cdot V^*(a\ ,\ o) + (1 - f)\cdot V^*(0\ ,\ 1) + U^*(1\ ,\ 1)$$

$$= fua + u(1 - f) \quad ,$$

and if $ua - 1 < 0$, $I = (1\ ,\ 1)$ belongs to the U decision region and the expected U gain function is of the form,

$$V^*(1\ ,\ 1) = f(1 - u)\cdot V^*(a\ ,\ 0) + (1 - f + uf)\cdot V^*(0\ ,\ 1) + U^*(1\ ,1)$$

$$= f(1 - u)\cdot ua + u \quad .$$

Then, we proceed to the point of $I = (2\ ,\ 1)$, where the delta function and the expected U gain function are written as

$$\delta(2\ ,\ 1) = V^*(a + 1,\ 0) - V^*(1\ ,\ 1) - 1 \quad ,$$

$$V^*(2\ ,\ 1) = t^*(2\ ,\ 1)\cdot V^*(a + 1,\ 0) + \overline{t}^*(2\ ,\ 1)\cdot V^*(1\ ,\ 1)$$

$$+ U^*(2\ ,\ 1) \quad .$$

---

1 Here we include the neutral decision point within the F decision region.

Since we have already calculated $V^*(a + 1, 0)$ and $V^*(1, 1)$, the sign of $\delta(2, 1)$ is easily obtained and so the U gain $V^*(2, 1)$.

Apply the same technique successively to $\delta(3, 1)$, $V^*(3, 1)$, $\delta(4, 1)$, and so on, until we cover some reasonably large range of $\underline{x}$ for $y = 1$. Then proceed to $y = 2$ applying the same technique. Thus, we will eventually cover a desired range of $\underline{x}$ and $\underline{y}$ within which the F- and U- regions and the values of $\underline{V^*}$ will be computed for the given set of parameters, $\underline{u}$, $\underline{f}$, and $\underline{a}$.

Usually, on each fixed value of $\underline{y}$ except $y = 0$, the sign of the delta function will be positive for relatively small values of $\underline{x}$, and then it turns to negative after passing the decision shifting point and will never be positive again. In other words $\delta(x, y)$ seems to be a monotone decreasing function of $\underline{x}$ for any $\underline{y}$. Although this hypothesis has not been proved, we are convinced of its truth and used it to simplify our computer program described later.

7. Some examples of actual computation

A DEC PDP-1 computer was used for our computation. Fig. 3 shows an example of the flow chart for the computation. Some examples of the results of computations for the delta function are shown in Fig. 4. The dotted line of each graph represents the border between the F decision region and the U decision region. In Fig. 5 are presented the values of $\underline{V^*}$ plotted against the internal decision context $\underline{x}$ for several values of $\underline{y}$.

8. A pilot experiment for G4

In this section, we report an experiment in which G4 was played by human subjects. The subjects were the staff's wives and secretaries of

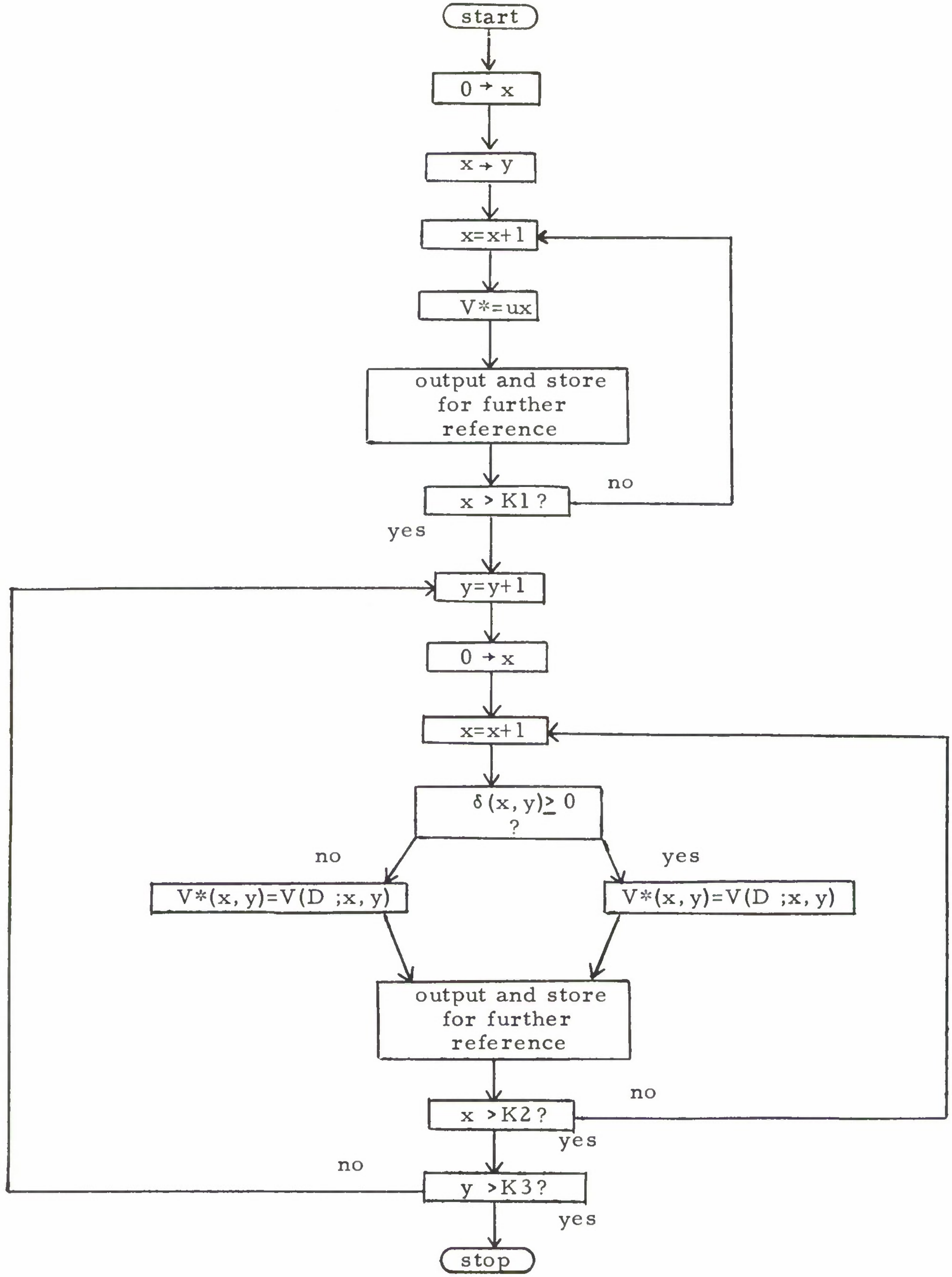

start
0 → x
x → y
x=x+1
V*=ux
output and store for further reference
x > K1 ?
no
yes
y=y+1
0 → x
x=x+1
δ (x, y) ≥ 0 ?
no
yes
V*(x, y)=V(D ;x, y)
V*(x, y)=V(D ;x, y)
output and store for further reference
x >K2 ?
no
yes
y >K3?
no
yes
stop

Fig. 3

FIG. 4

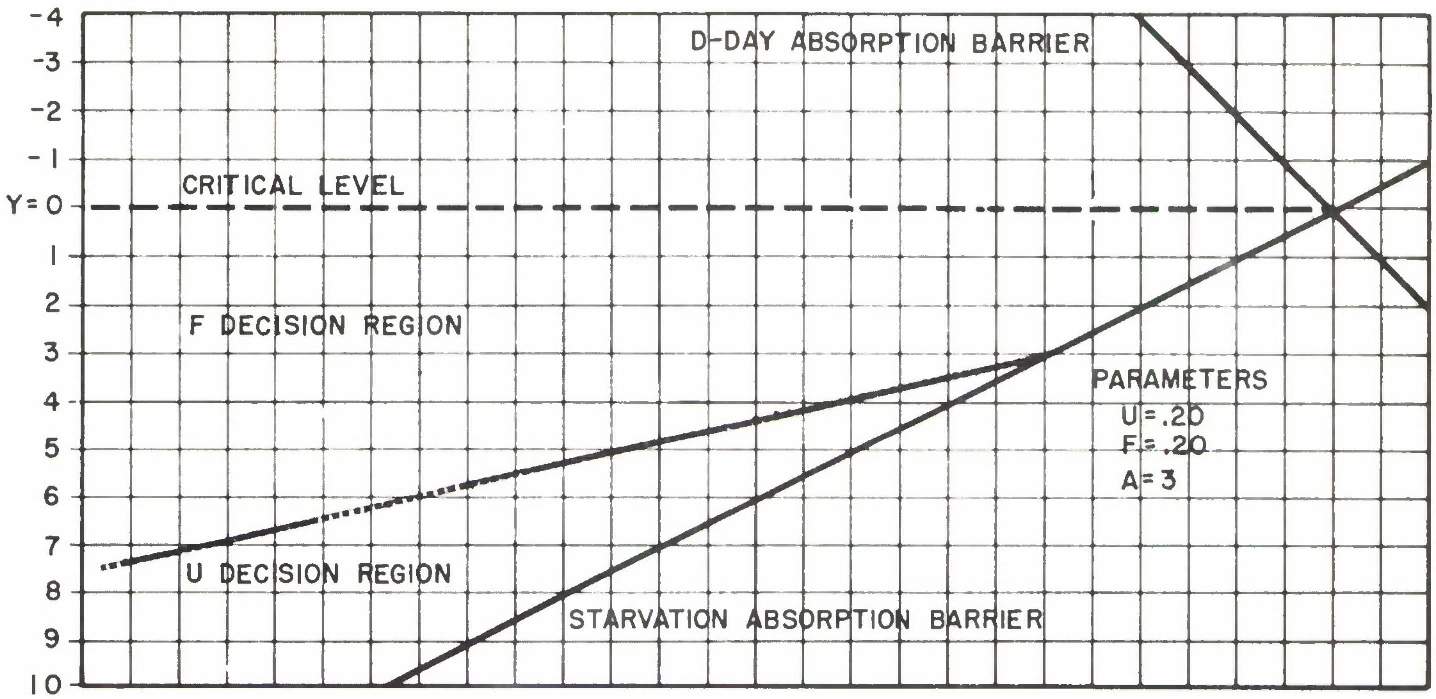

-4
-3
-2
-1
Y=0
1
2
3
4
5
6
7
8
9
10
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
F DECISION REGION
PARAMETERS
U=.20
F=.20
A=3
U DECISION REGION
STARVATION ABSORPTION BARRIER

FIG. 4A

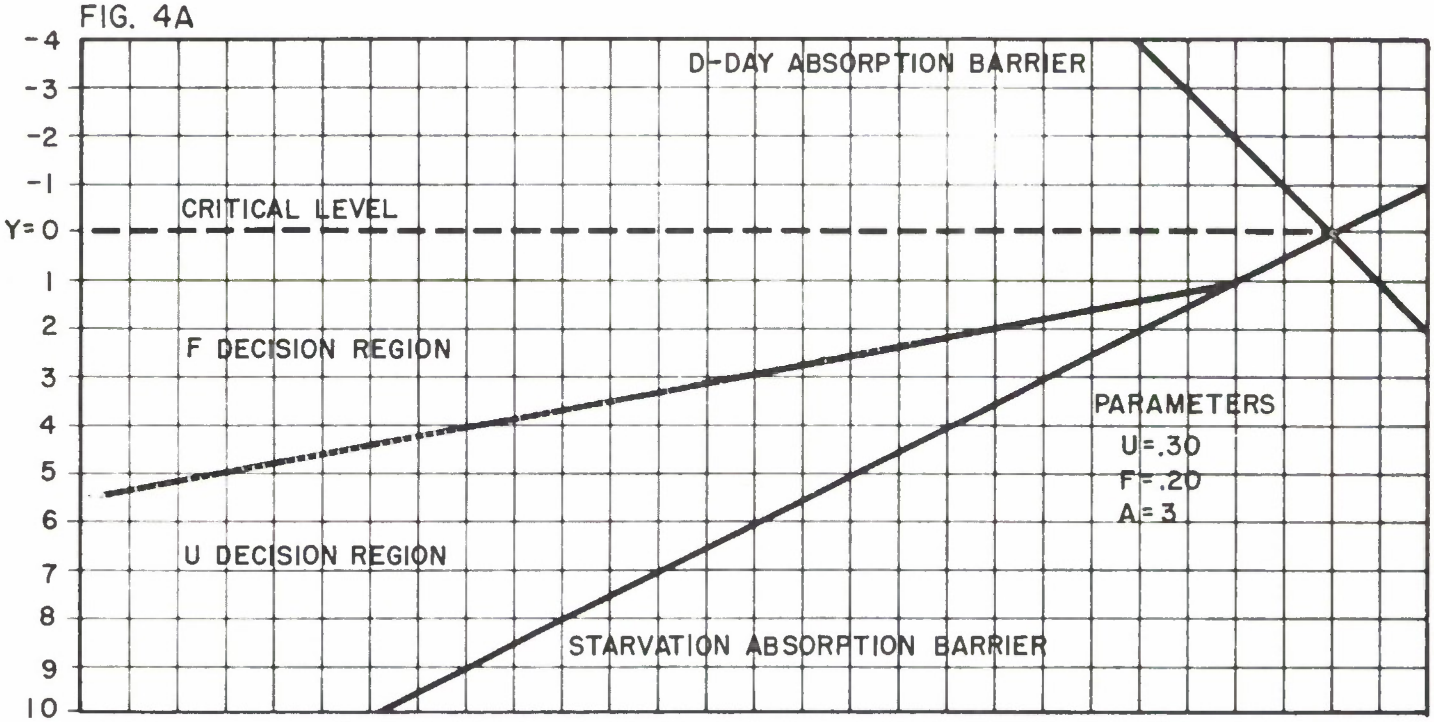
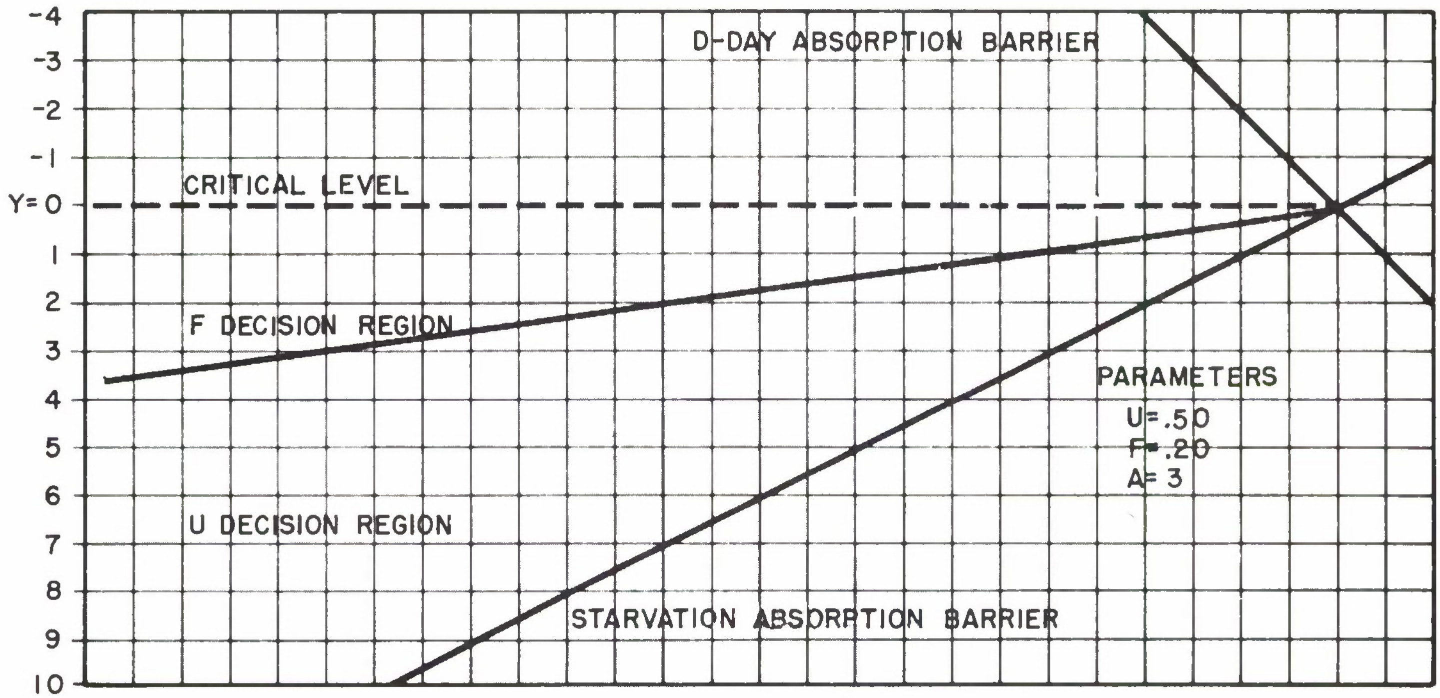

-4
-3
-2
-1
Y=0
1
2
3
4
5
6
7
8
9
10
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
F DECISION REGION
PARAMETERS
U=.30
F=.20
A=3
U DECISION REGION
STARVATION ABSORPTION BARRIER

FIG. 4B



-4
-3
-2
-1
Y= 0
1
2
3
4
5
6
7
8
9
10
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
F DECISION REGION
PARAMETERS
U=.50
F=.20
A= 3
U DECISION REGION
STARVATION ABSORPTION BARRIER

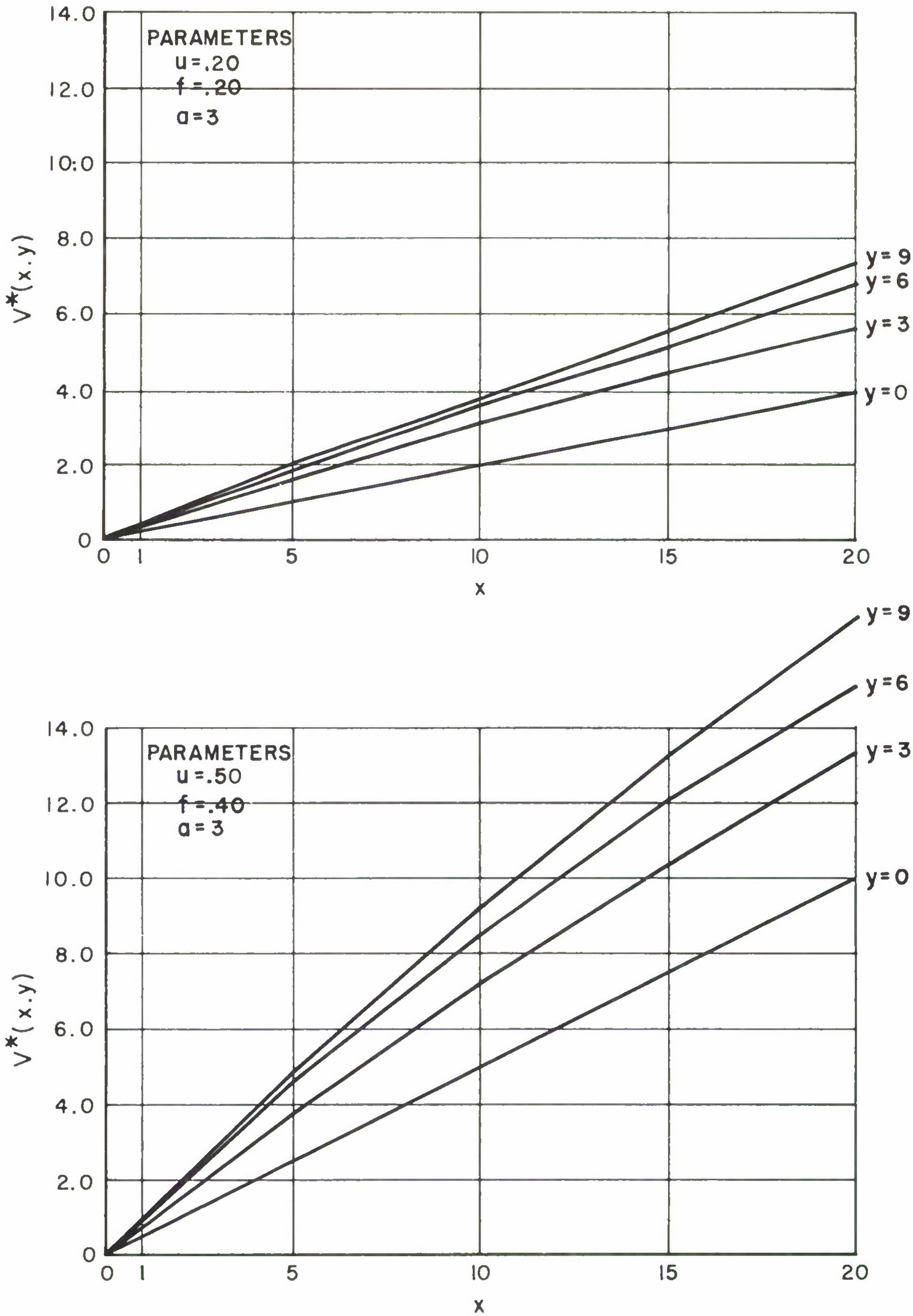

PARAMETERS
u=.20
f=.20
a=3
V*(x.y)
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0
0
1
5
10
15
20
x
y=9
y=6
y=3
y=0
PARAMETERS
u=.50
f=.40
a=3
V*(x.y)
y=9
y=6
y=3
y=0

FIG. 5

Institute for Research, State College, Pennsylvania. All the subjects being gathered in a single room were taught the rules of G4 including the values of the parameters, $\underline{u}$, $\underline{f}$, and $\underline{a}$, which were different from session to session. On each session an IC diagram sheet was given to each subject on which were drawn the starvation absorption barrier and the Dooms Day barrier.

Before the presentation of the external decision context a "predecision" was required to be made on each trial. Here a "predecision" means the decision to be made prior to the presentation of the actual external decision context and supposing the external decision context as $z = (F\ U)$, namely the decision which subject will make if the external decision context turns out to be the non-trivial decision context. $\underline{S}$s were also required to make the predecisions for the future choice points, the choice points where he might possibly be in future, as many as possible.

When the predecisions were made, the experimenter threw two dice, one (twenty-sided) for the probability $\underline{f}$ and the other (ten-sided) for the probability $\underline{u}$, and announced the external decision context realized on that trial. It was required that the actual decisions to be made after the announcement be same as the predecision if the trial turned out to be a decision trial. If it turned out not to be the decision trial, $\underline{S}$s were not restrained to their predecisions as the case of a decision trial. They might take one uranium or one fungus according to the realized external decision context or they might just proceed one step to the starvation line if the external decision context was $z = (O\ O)$.

Depending upon the real decision made, $\underline{S}$s moved to the next choice point on their IC diagram sheet, and recorded their new predecisions there on the sheets. The subjects were requested to attempt to make their U return maximum at the end of each session. They were paid a quarter for each uranium they collected.

The values of the parameters and the results are shown in the following pages. Traces of Ss' actual locomotion are plotted on the IC diagrams and the corresponding optimal decision shifting line are given so that we can compare Ss' decisions with the optimal decisions. Double lines show that the trial which happened between two choice points was a decision trial.

Each circle above the locomotion line shows that S gained a piece of uranium there. Subject's predecisions were also plotted on the IC diagram, but we did not show all of them except the predecision made just for the next trial. Letters F and U on the locomotion lines are predecisions made by Ss.

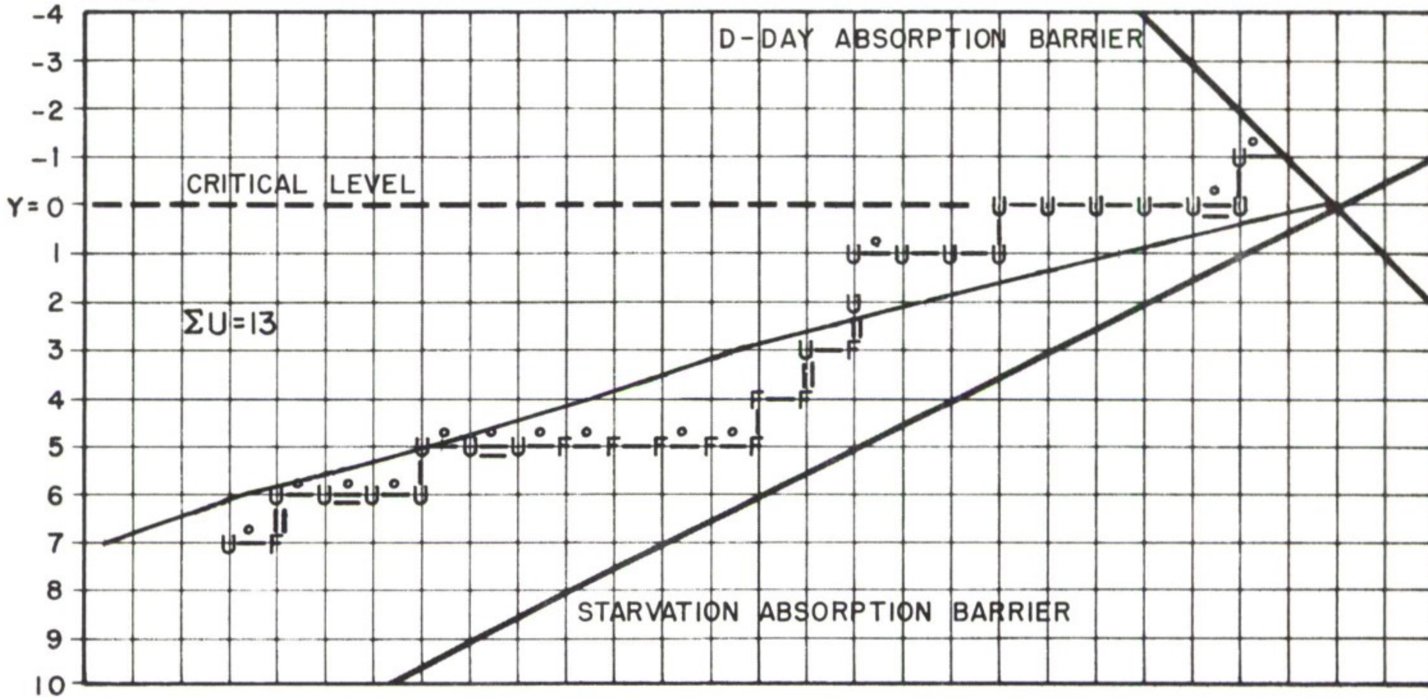
Session I

Values of parameters (u=.5 f=.4 a=3) : Starting conditions (x=9 y=7 L=31)

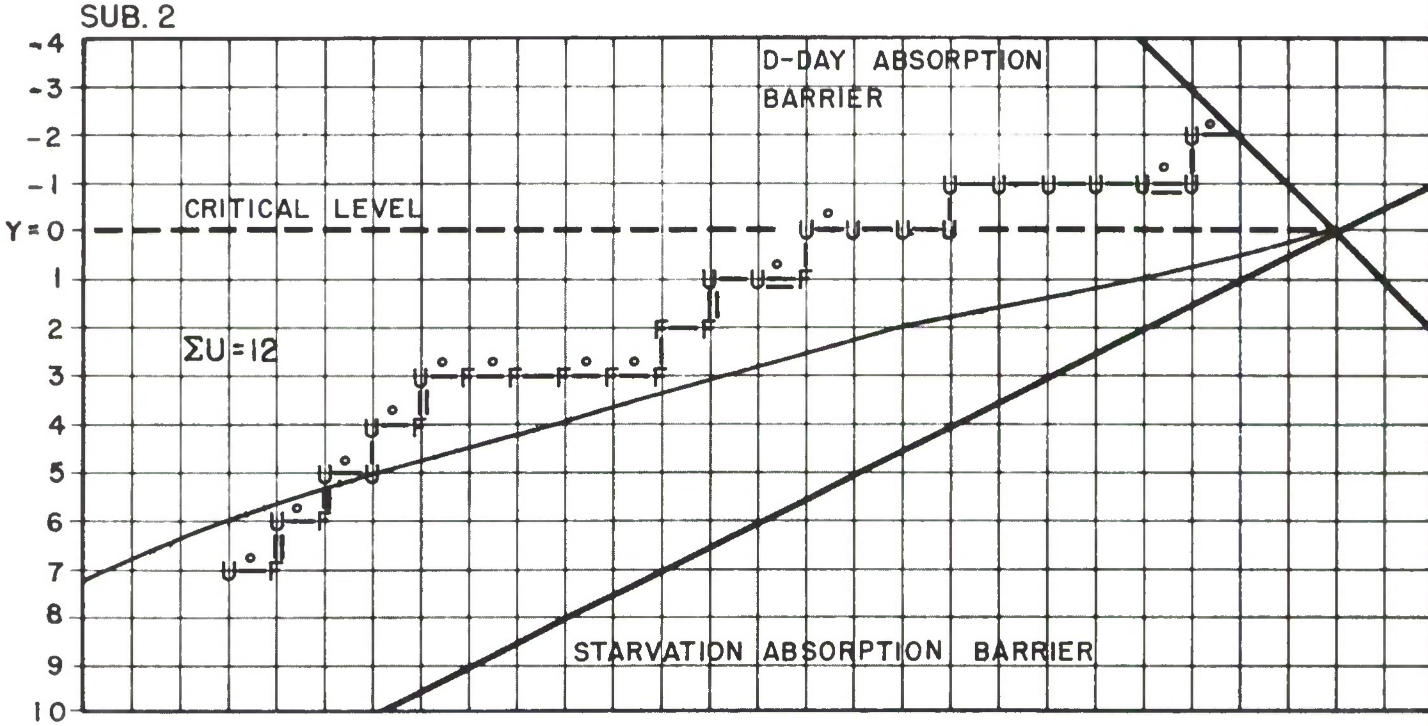
trial	outcome	trial	outcome
1	(U O)	16	(U F)
2	(U F)	17	(O O)
3	(U O)	18	(U F)
4	(U F)	19	(O F)
5	(U O)	20	(U O)
6	(O F)	21	(O O)
7	(U O)	22	(O O)
8	(U F)	23	(O F)
9	(U O)	24	(O O)
10	(U O)	25	(O O)
11	(O O)	26	(O O)
12	(U O)	27	(O O)
13	(U O)	28	(U F)
14	(O F)	29	(O F)
15	(O O)	30	(U O)

SESSION I

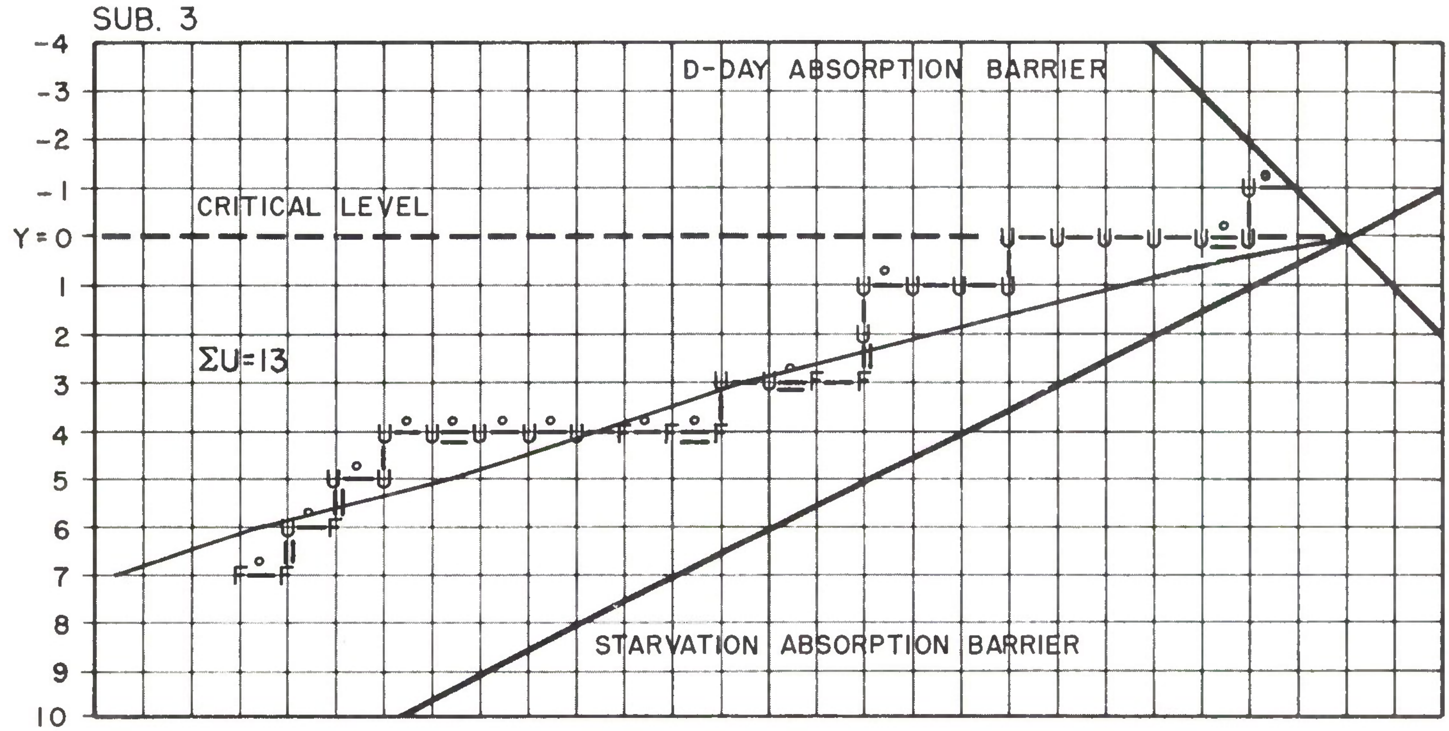
SUB. 1

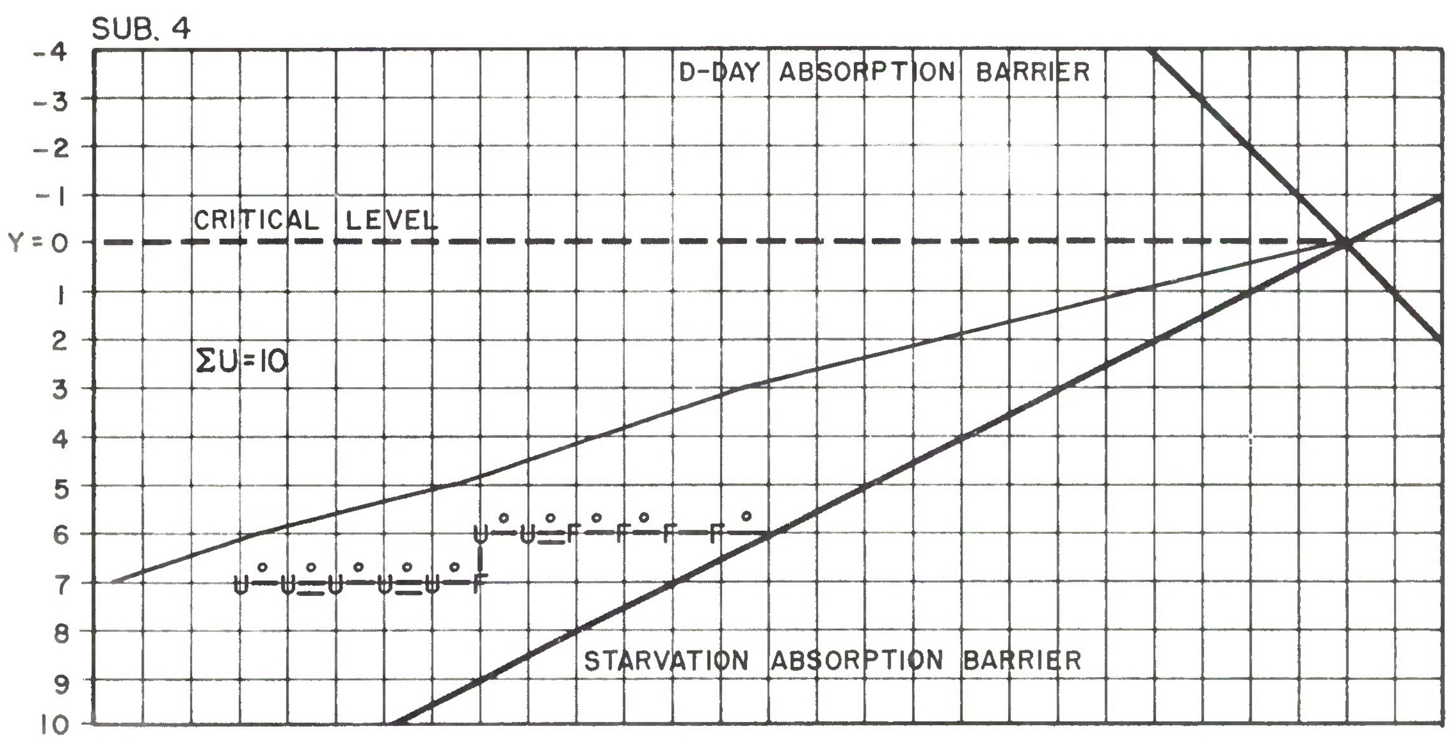


SUB. 2

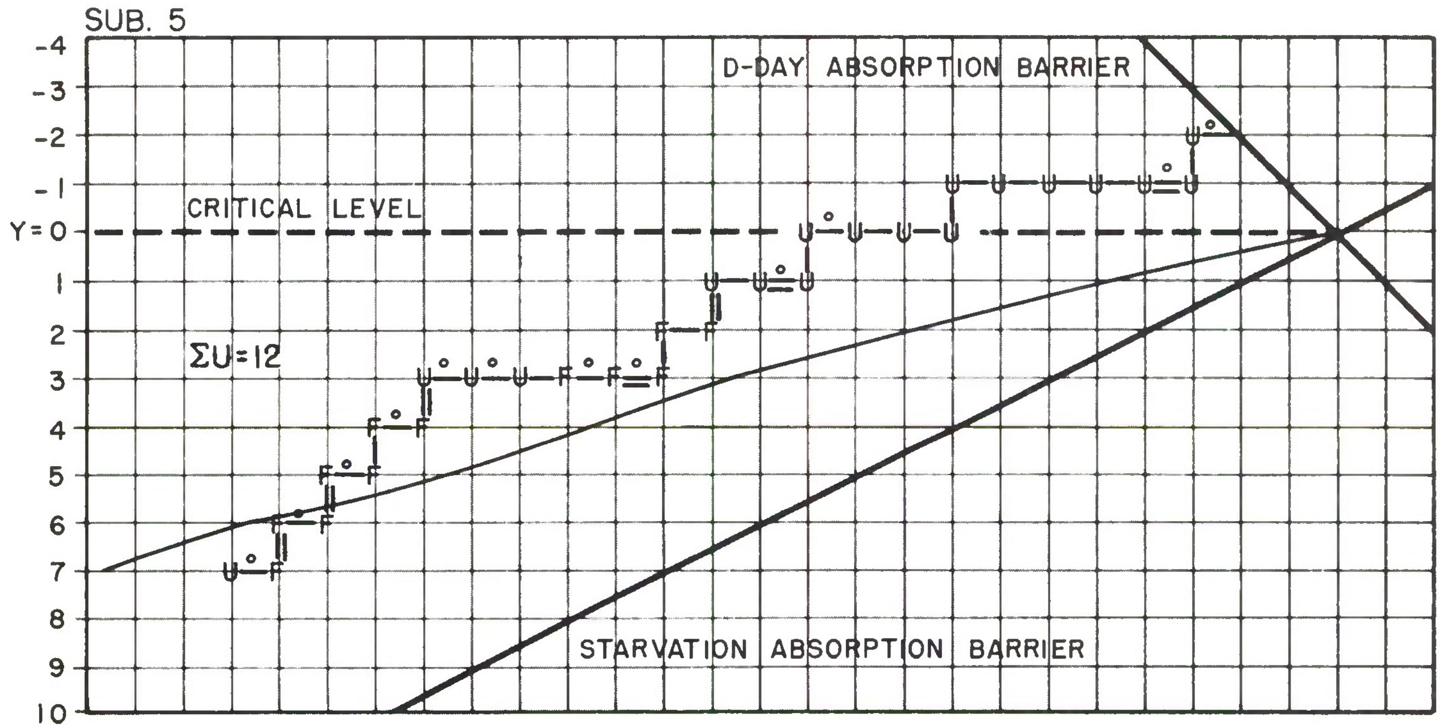


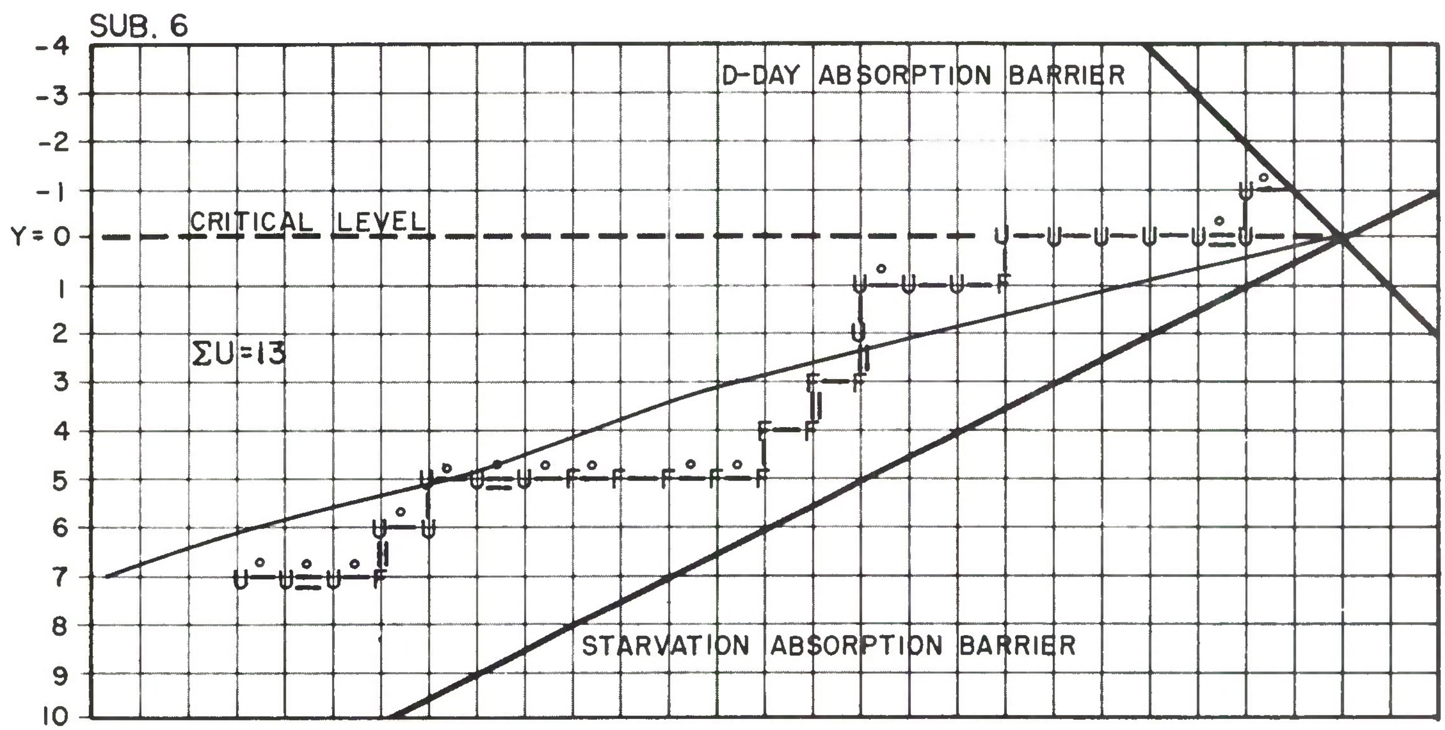
SESSION I (CONT.)





SESSION I (CONT.)



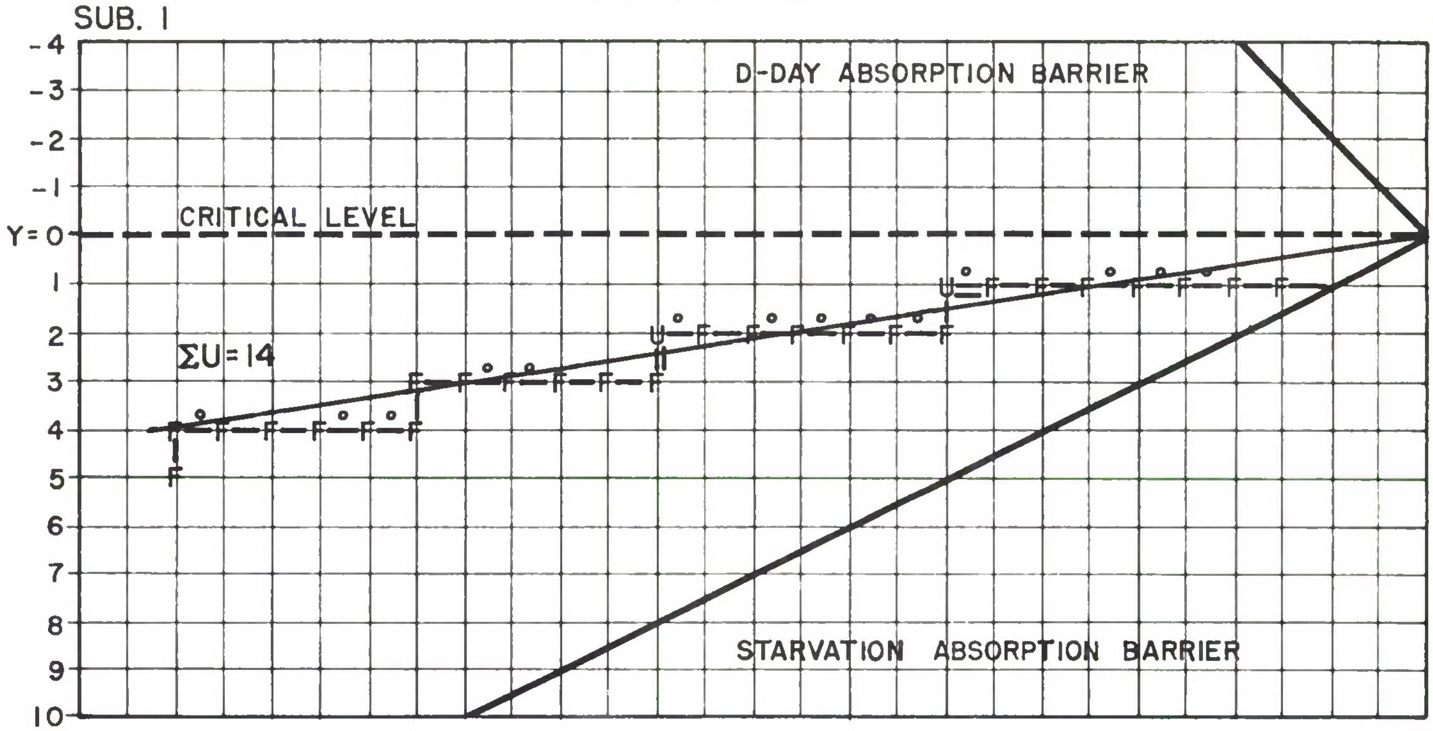


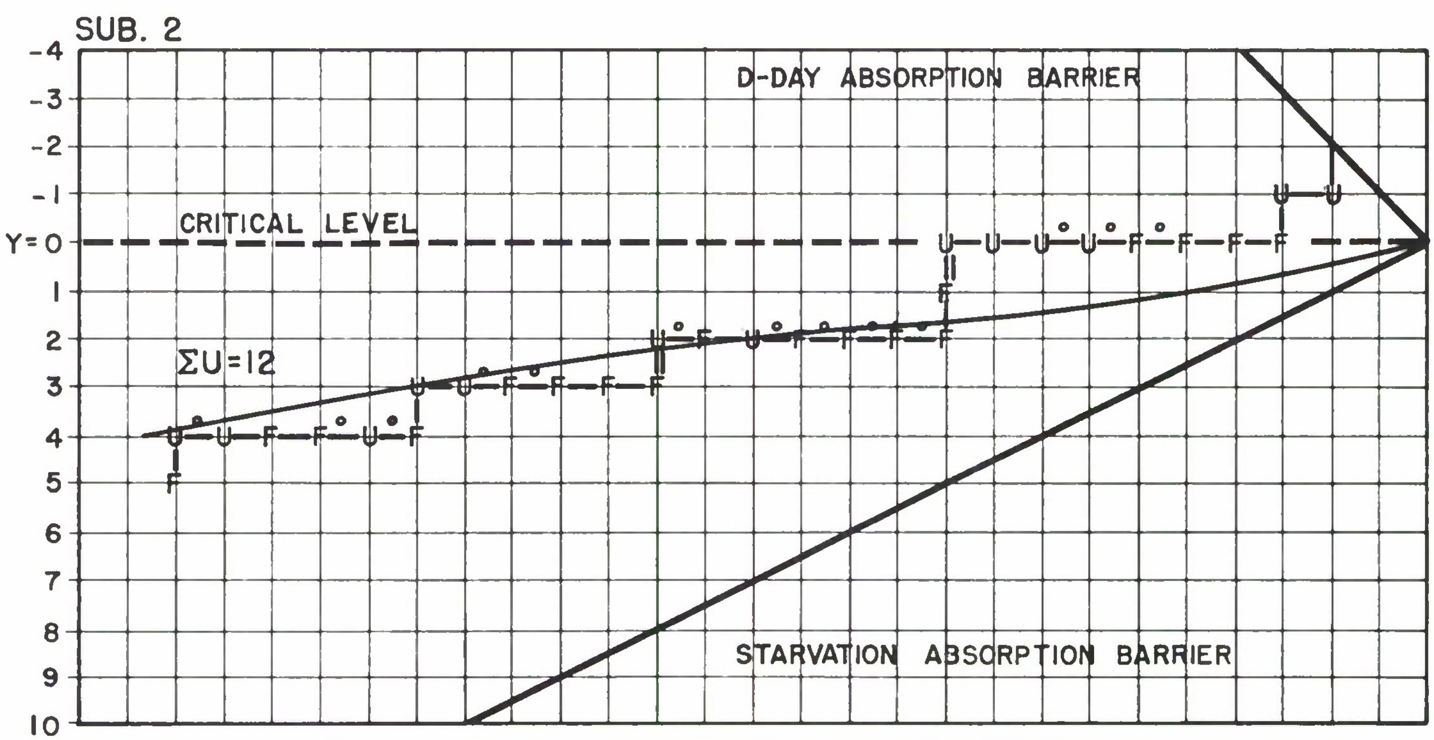
Session II

Values of parameters (u=.5 f=.2 a=3) ; Starting conditions (x=16 y=5 L=31)

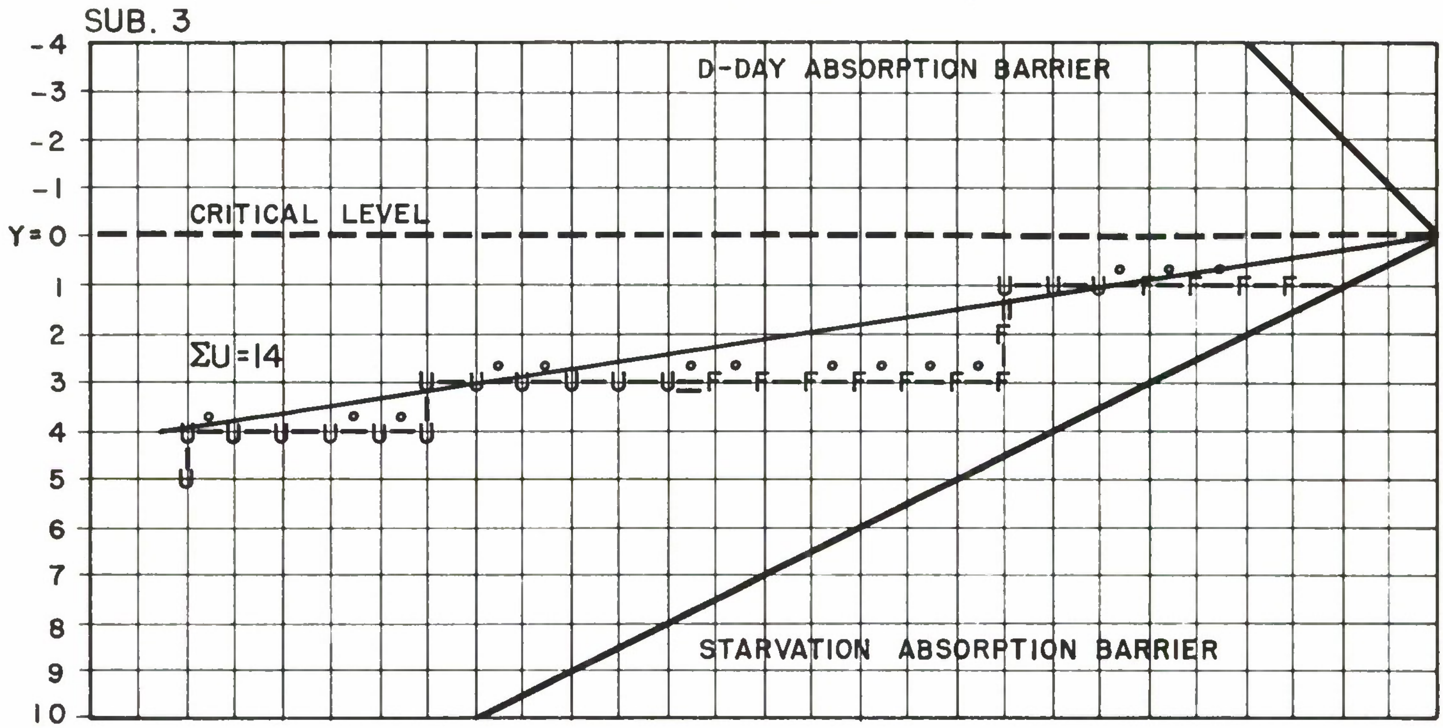
trial	outcome	trial	outcome
1	(O F)	16	(U O)
2	(U O)	17	(U O)
3	(O O)	18	(U O)
4	(O O)	19	(U O)
5	(U O)	20	(O F)
6	(U O)	21	(U F)
7	(O F)	22	(O O)
8	(O O)	23	(O O)
9	(U O)	24	(U O)
10	(U O)	25	(U O)
11	(O O)	26	(U O)
12	(O O)	27	(O O)
13	(U F)	28	(O O)
14	(U O)	29	(O F)
15	(O O)	30	(O O)
		31	(O F)

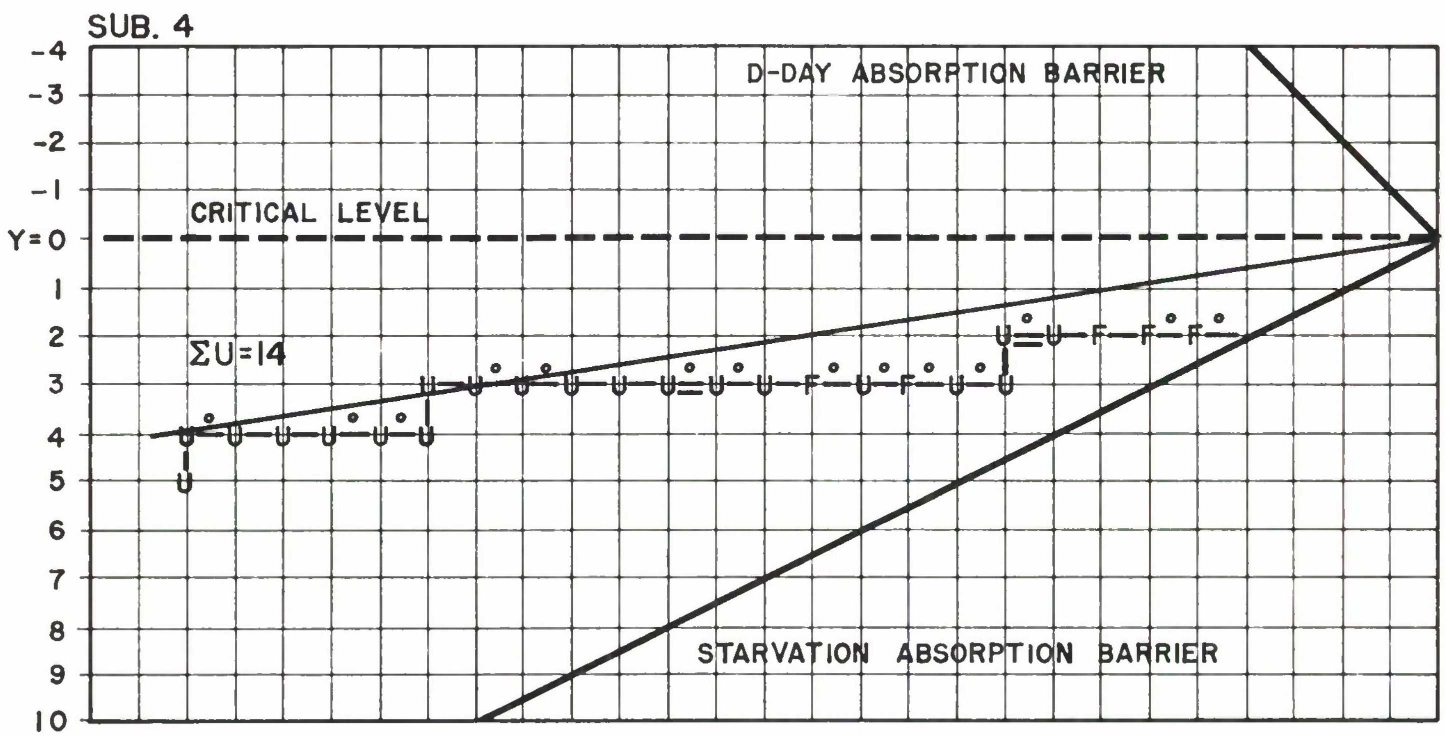
SESSION 2


SUB. 1
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y=0
ΣU=14
STARVATION ABSORPTION BARRIER

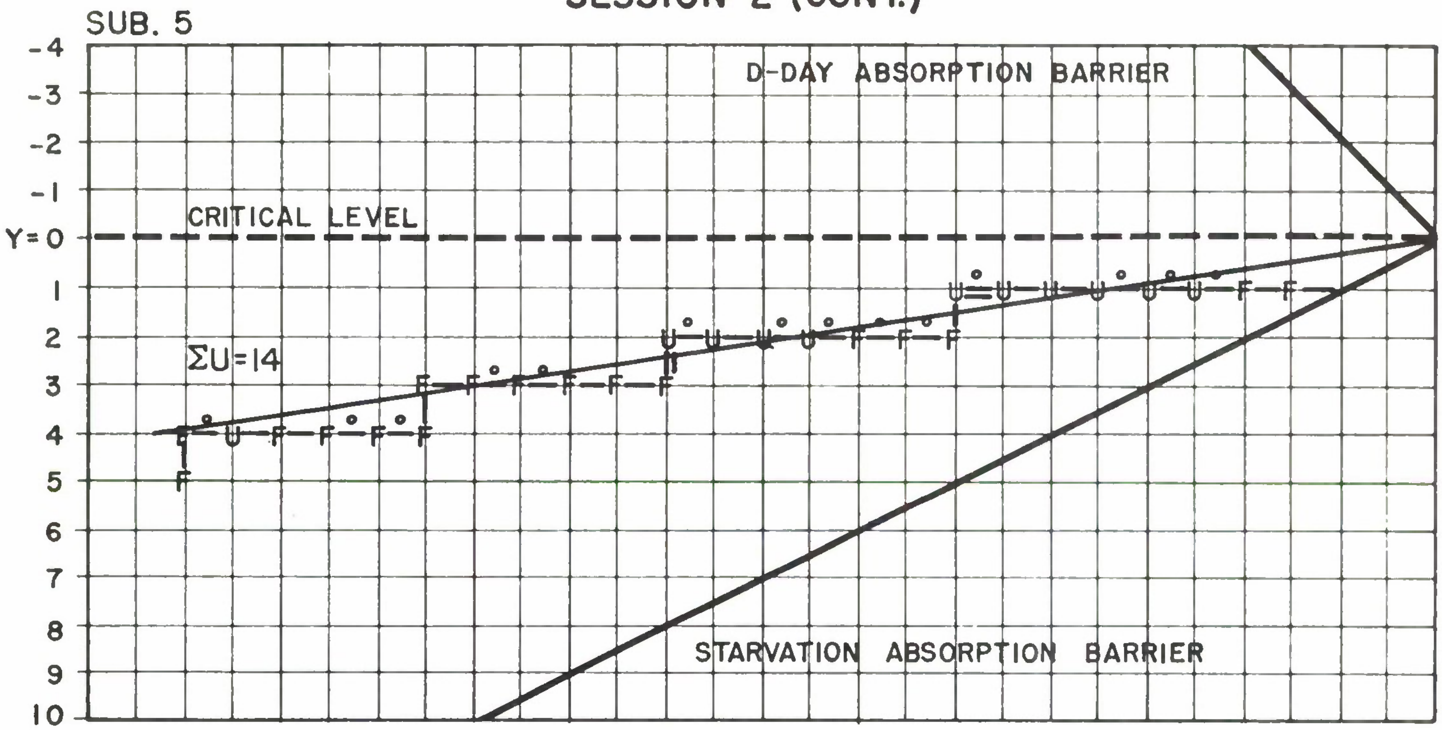

SUB. 2
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y=0
ΣU=12
STARVATION ABSORPTION BARRIER

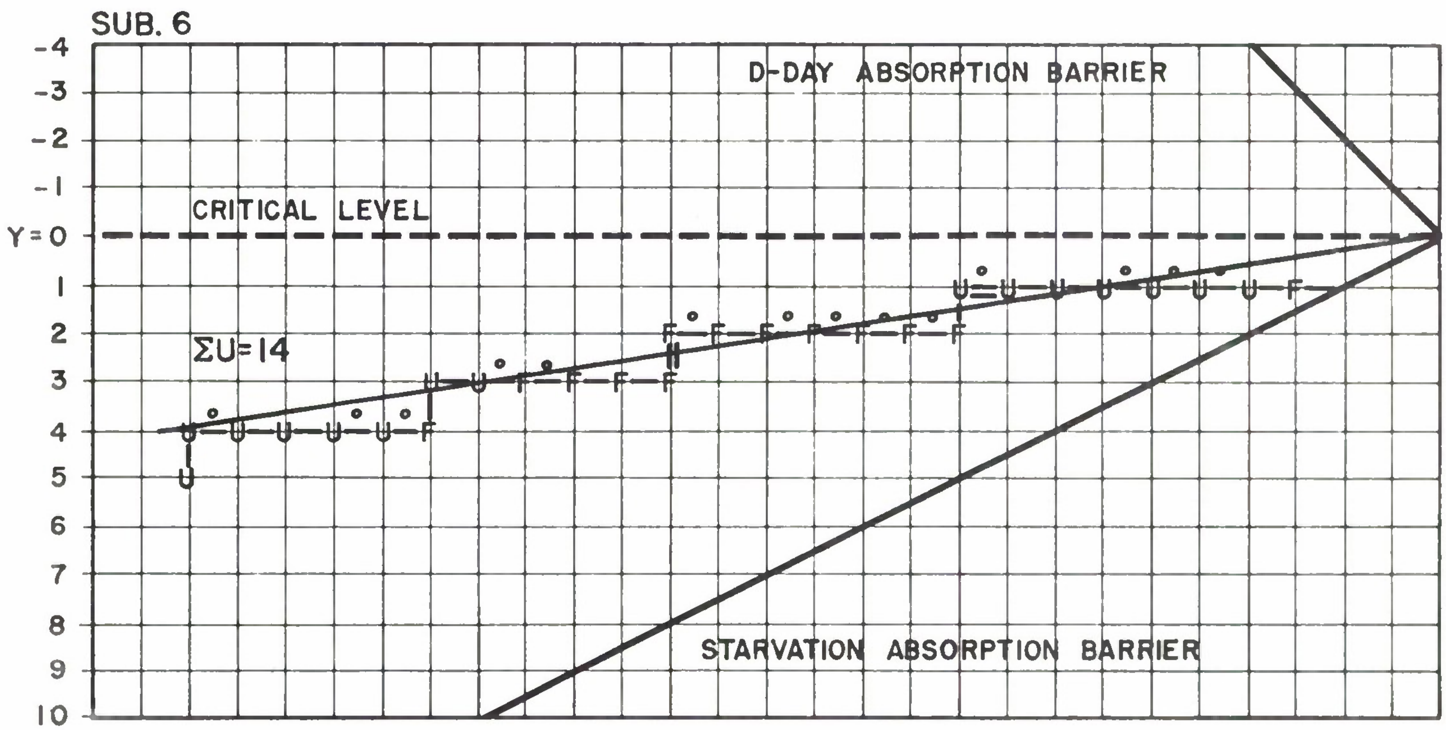
SESSION 2 (CONT.)





SESSION 2 (CONT.)



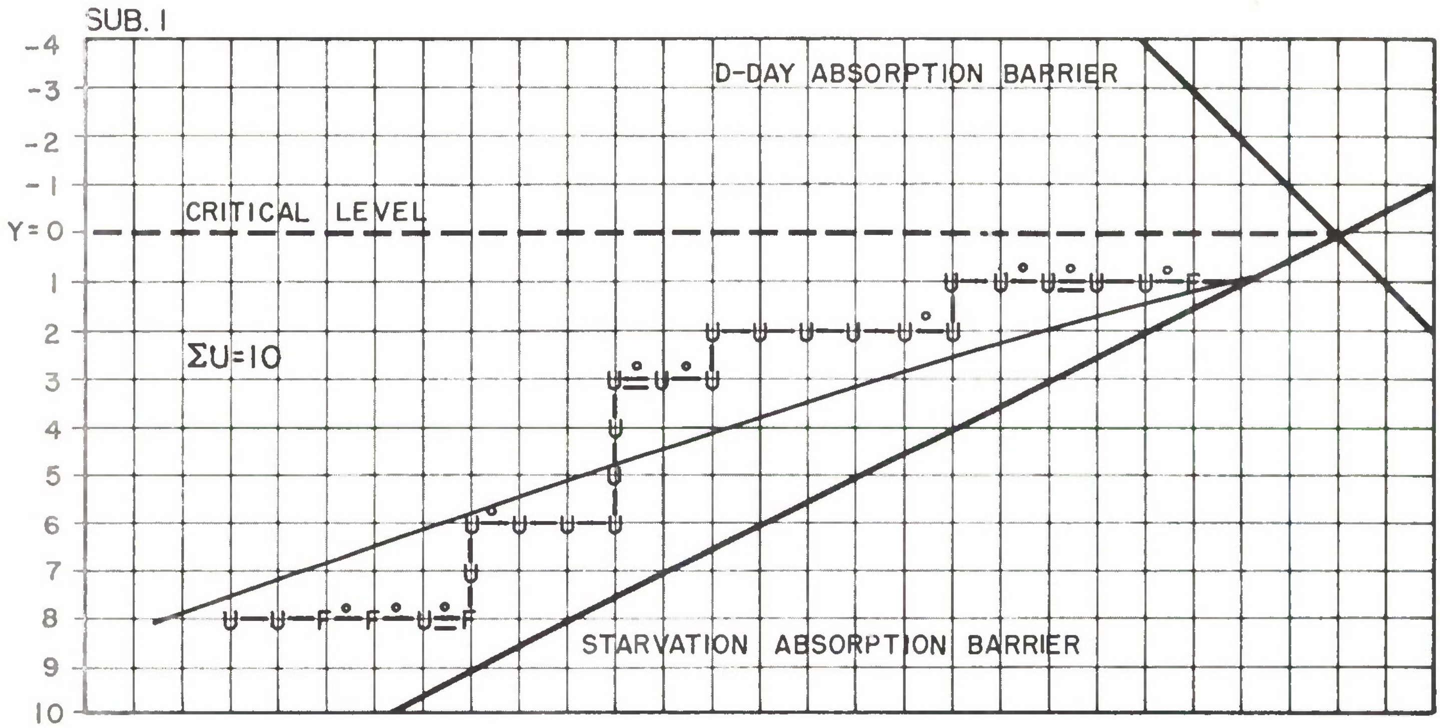


Session III

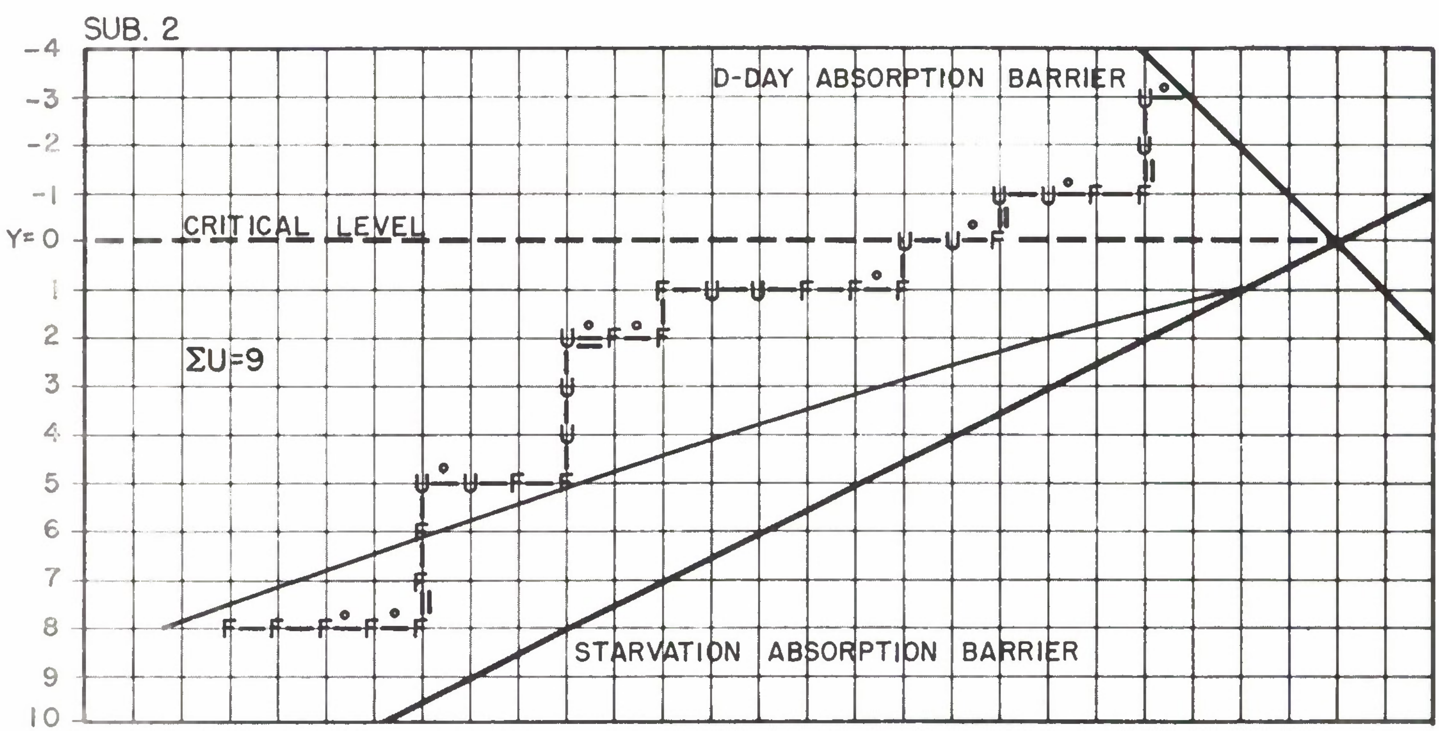
Values of parameters (u=.3 f=.4 a=3) : Starting conditions (x=7 y=8 L=31)

trial	outcome	trial	outcome
1	(O O)	16	(O F)
2	(O O)	17	(O O)
3	(U O)	18	(O O)
4	(U O)	19	(O O)
5	(U F)	20	(O O)
6	(O F)	21	(U O)
7	(O F)	22	(O F)
8	(U O)	23	(O O)
9	(O O)	24	(U O)
10	(O O)	25	(U F)
11	(O F)	26	(O O)
12	(O F)	27	(U O)
13	(O F)	28	(O O)
14	(U F)	29	(U F)
15	(U O)	30	(O F)
		31	(U O)

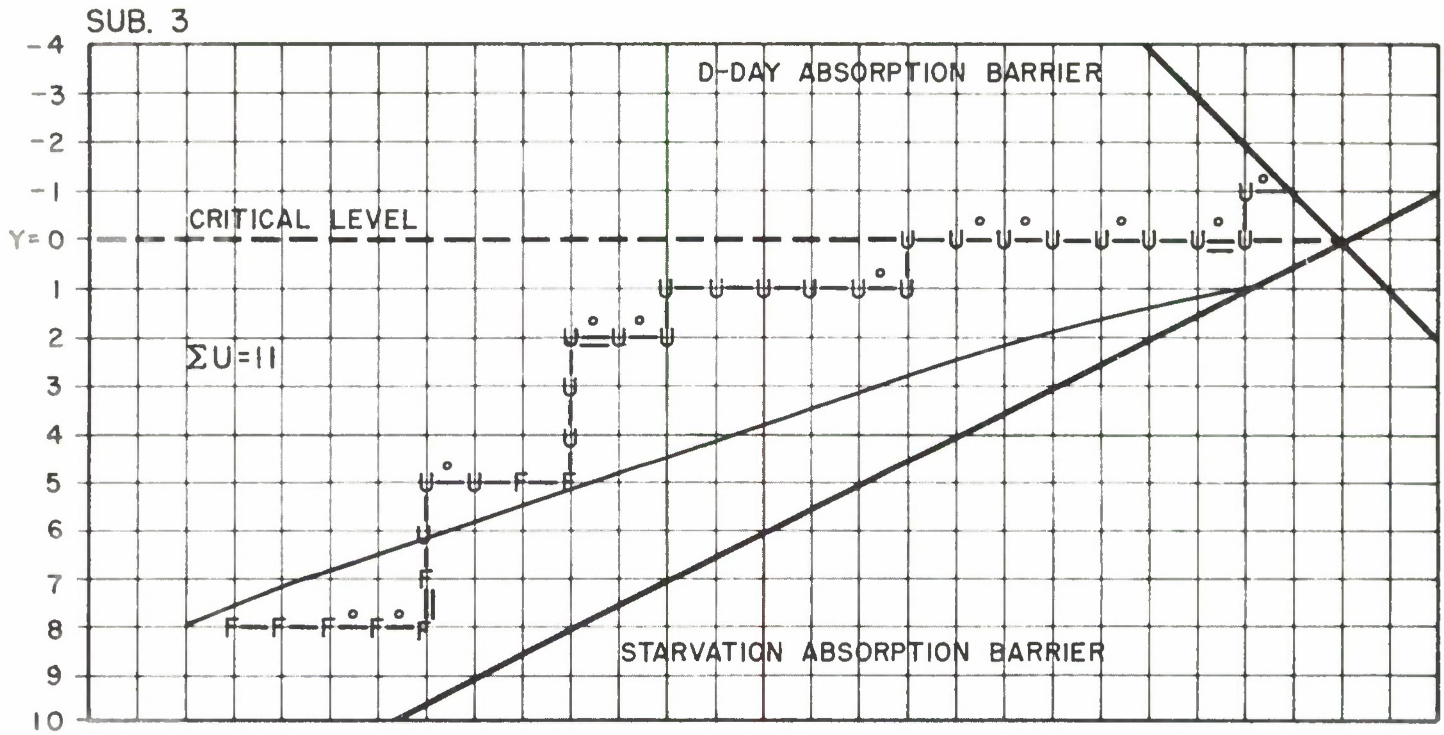
SESSION 3

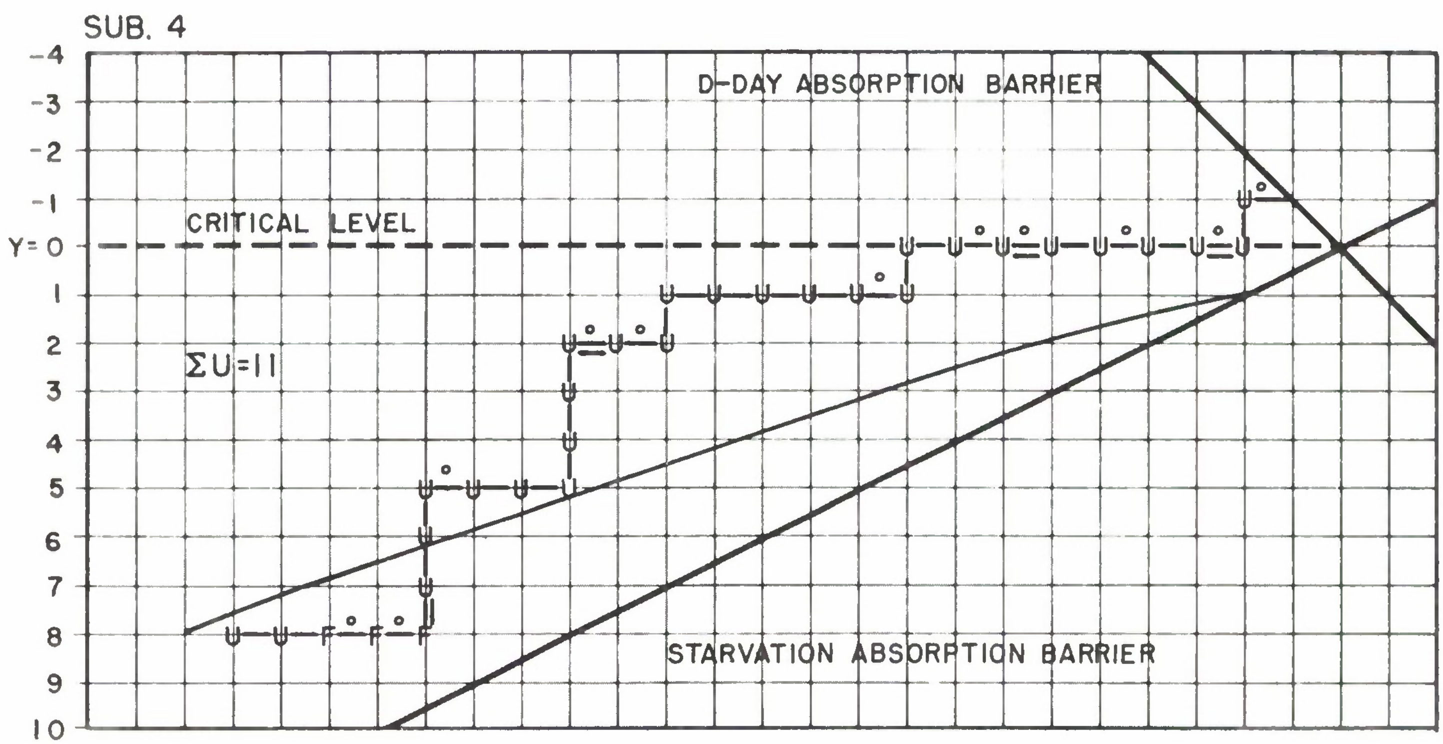


SUB. 1
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y = 0
ΣU=10
STARVATION ABSORPTION BARRIER

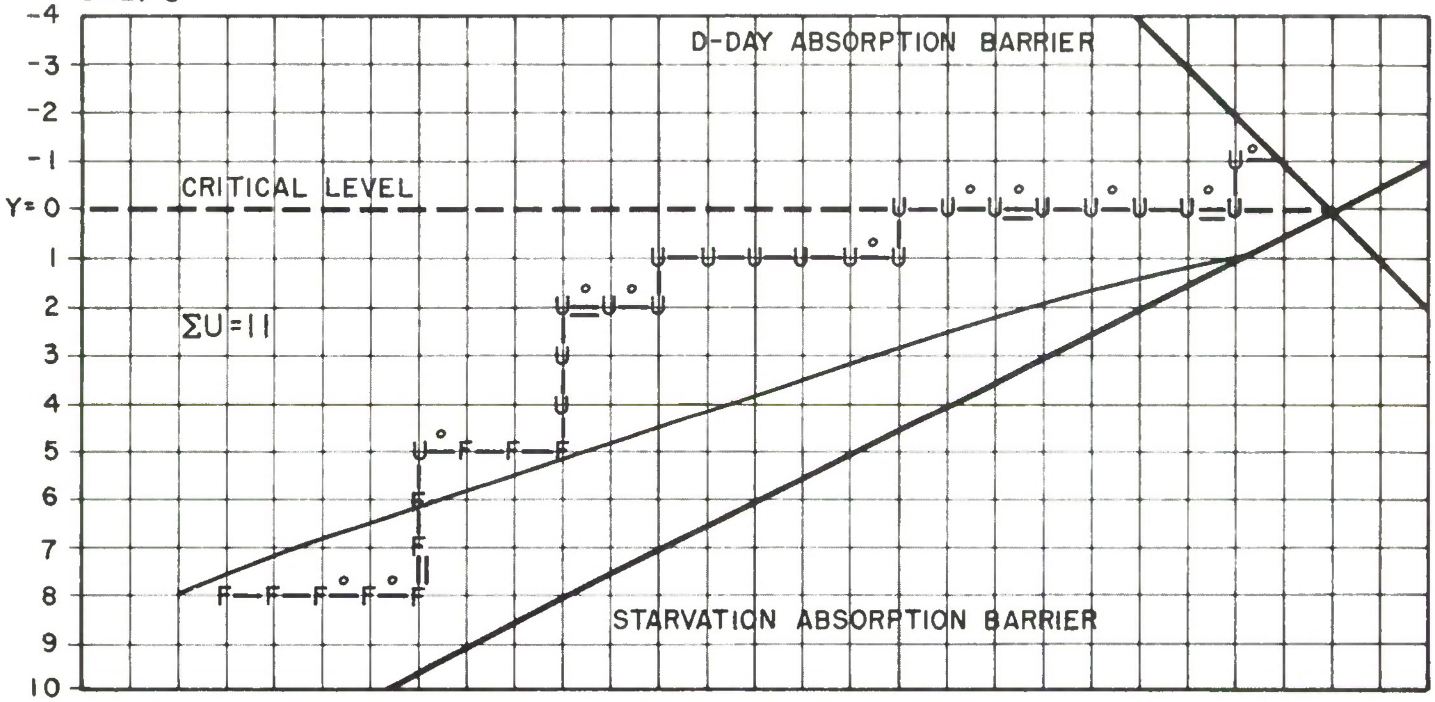


SUB. 2
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y = 0
ΣU=9
STARVATION ABSORPTION BARRIER

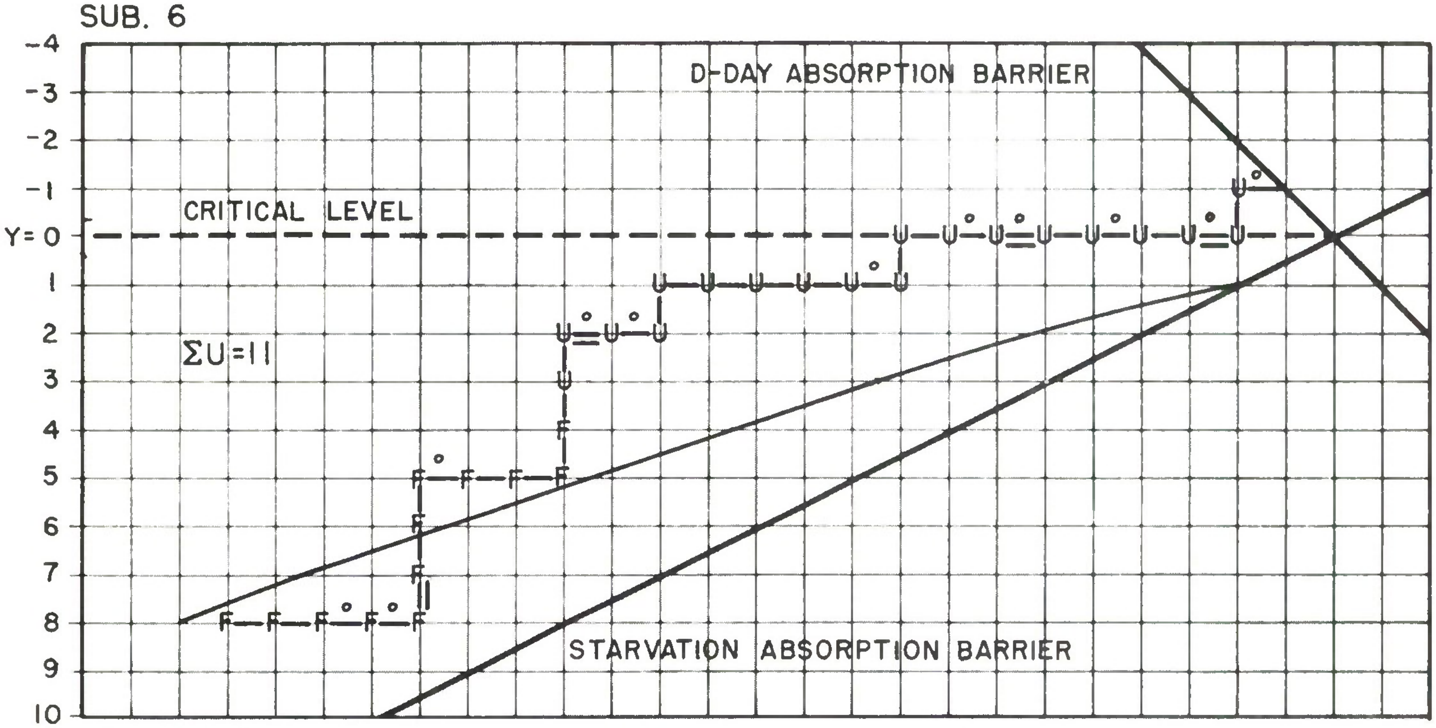

SUB. 3
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y= 0
ΣU=11
STARVATION ABSORPTION BARRIER
-4
-3
-2
-1
1
2
3
4
5
6
7
8
9
10


SUB. 4
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y= 0
ΣU=11
STARVATION ABSORPTION BARRIER
-4
-3
-2
-1
1
2
3
4
5
6
7
8
9
10

SUB. 5


D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y= 0
ΣU=11
STARVATION ABSORPTION BARRIER

SUB. 6

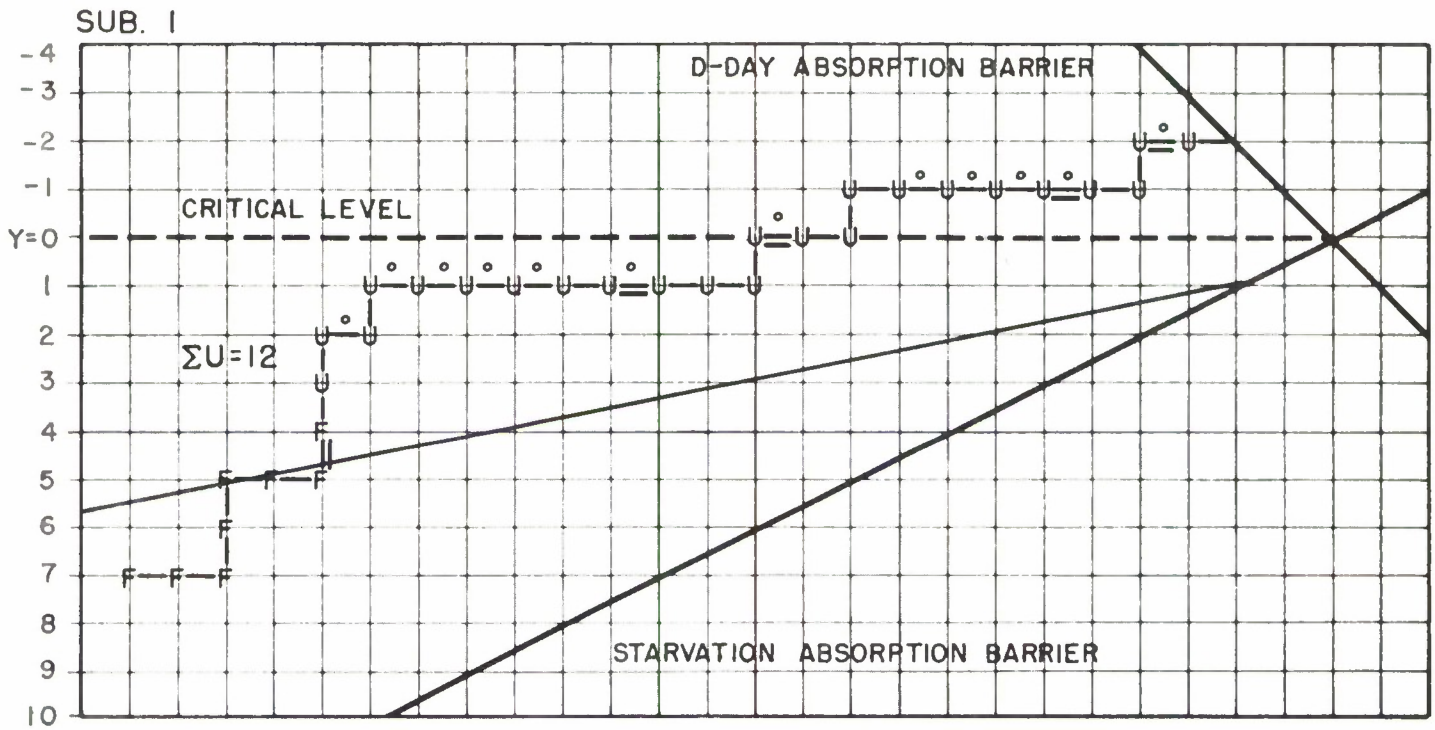

D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y= 0
ΣU=11
STARVATION ABSORPTION BARRIER

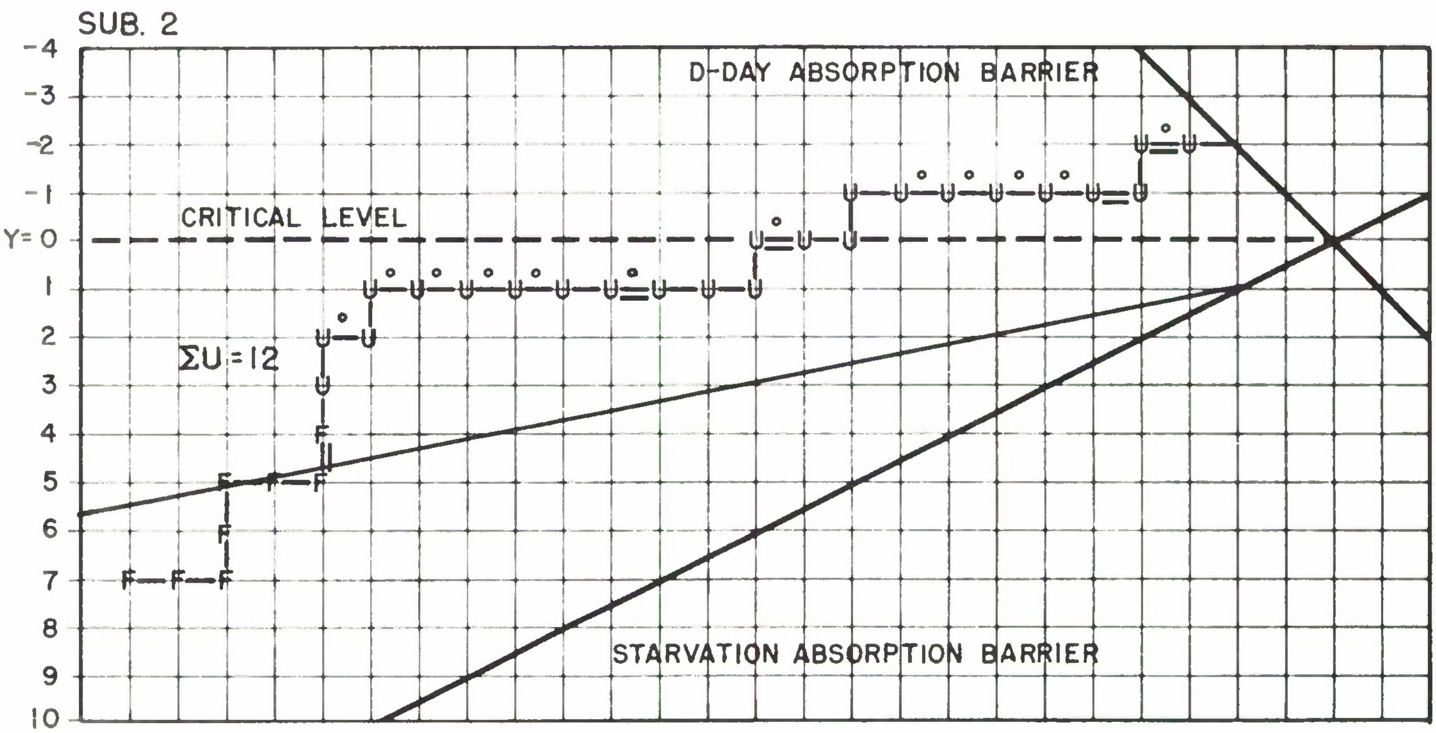
Session IV A

Values of parameters (u=. 3 f+. 2 a=3) : Starting conditions (x=11 y=7 L=32)

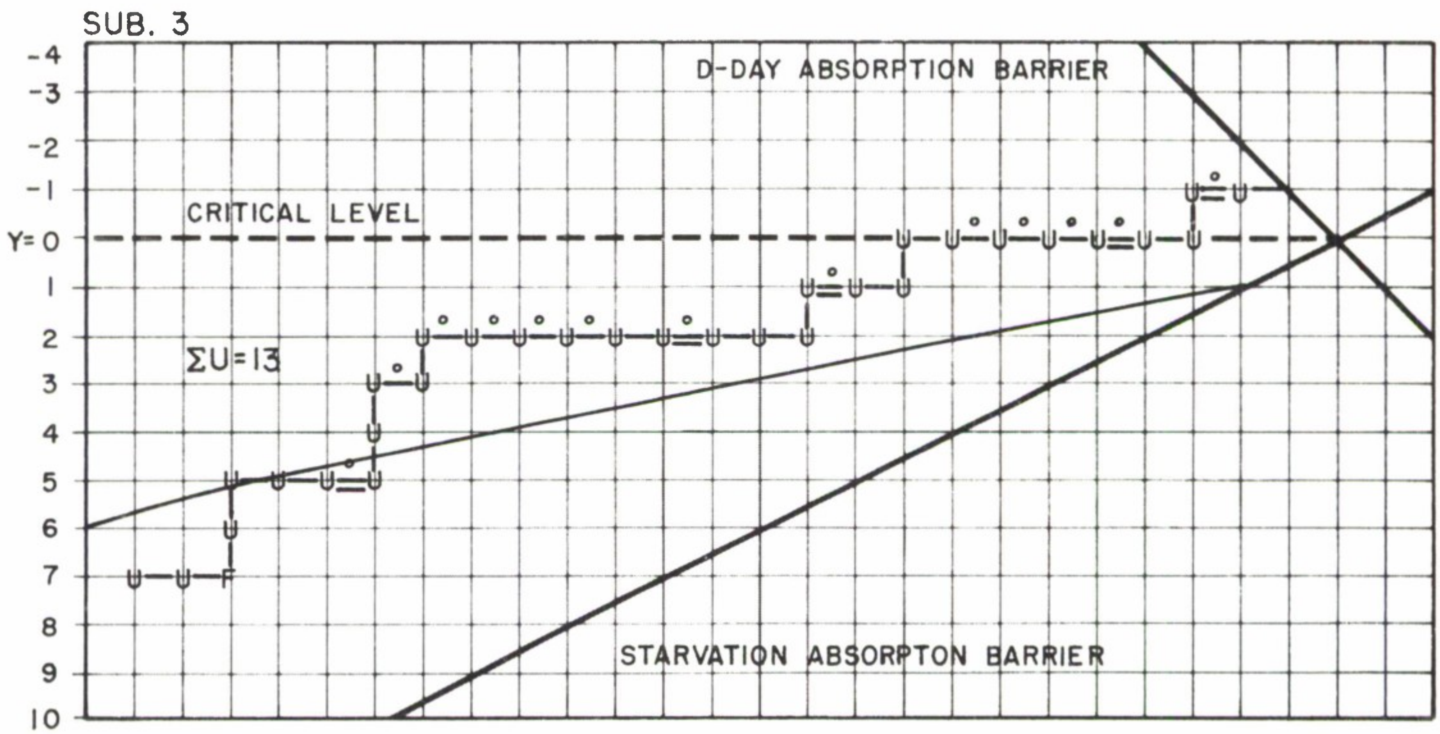
trial	outcome	trial	outcome
1	(O O)	17	(U F)
2	(O O)	18	(O O)
3	(O F)	19	(O O)
4	(O F)	20	(O F)
5	(O O)	21	(U F)
6	(O O)	22	(O O)
7	(U F)	23	(O F)
8	(O F)	24	(O O)
9	(O F)	25	(U O)
10	(U O)	26	(U O)
11	(O F)	27	(U O)
12	(U O)	28	(U F)
13	(U O)	29	(O O)
14	(U O)	30	(O F)
15	(U O)	31	(U F)
16	(O O)	32	(O O)

SESSION 4A


SUB. 1
-4
-3
-2
-1
Y=0
1
2
3
4
5
6
7
8
9
10
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
ΣU=12
STARVATION ABSORPTION BARRIER


SUB. 2
-4
-3
-2
-1
Y=0
1
2
3
4
5
6
7
8
9
10
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
ΣU=12
STARVATION ABSORPTION BARRIER

SESSION 4A (CONT.)

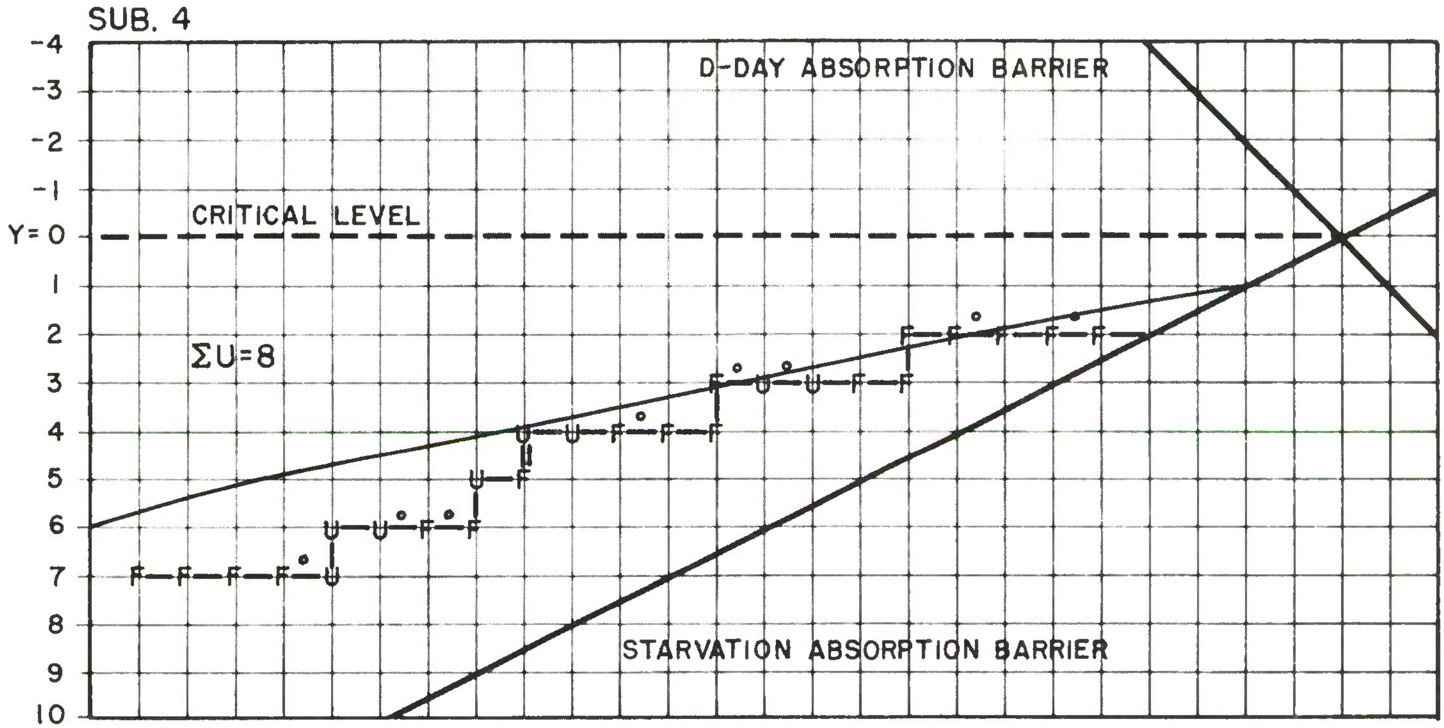


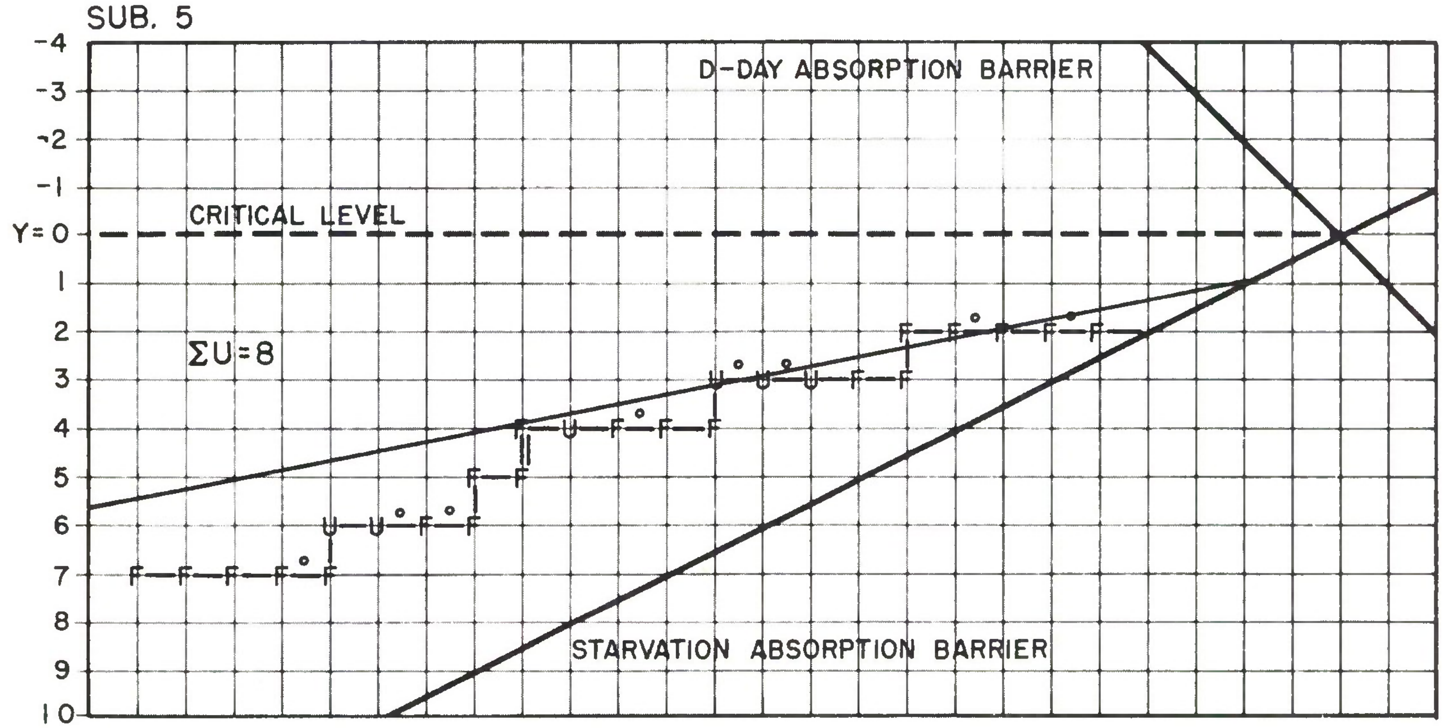
Session IV B

Values of parameters (u=.3 f=.2 a=3) : Starting conditions (c=11 y=7 L=32)

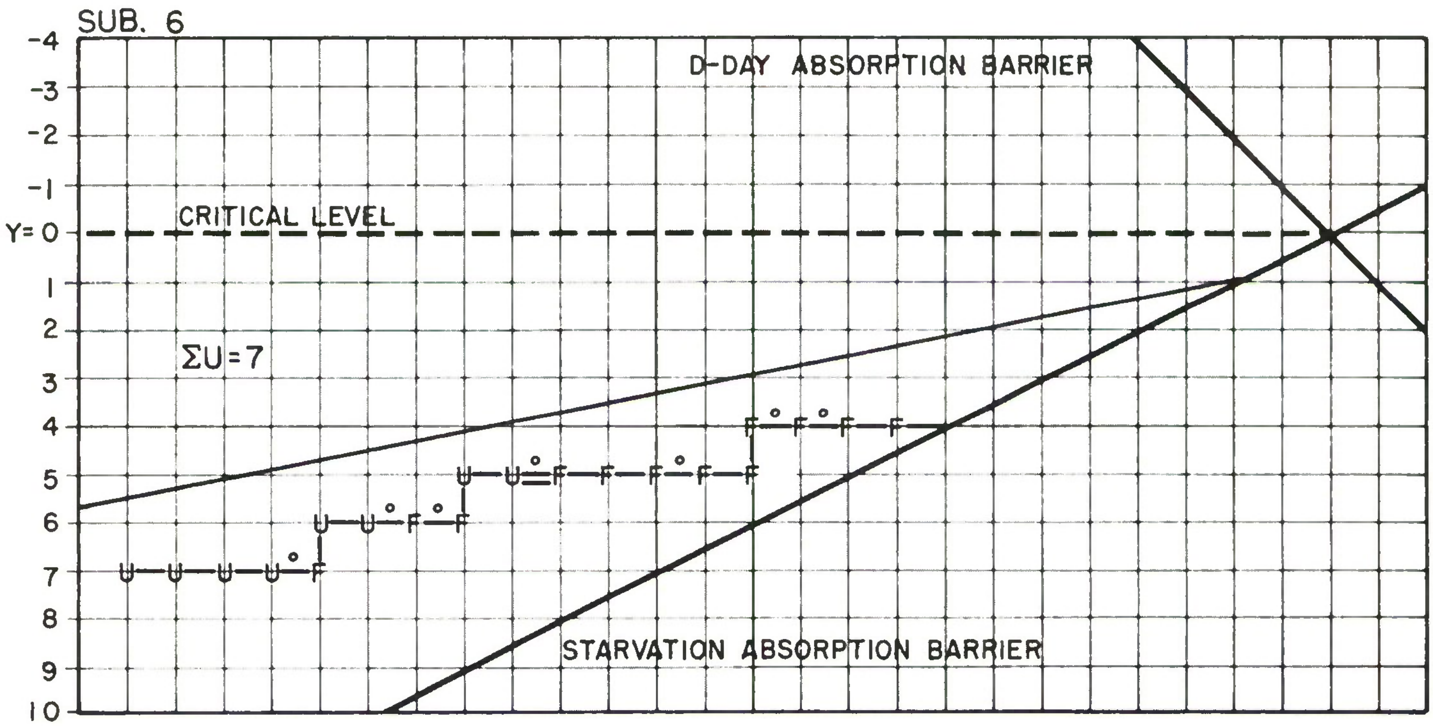
trial	outcome	trial	outcome
1	(O O)	14	(U O)
2	(O O)	15	(O O)
3	(O O)	16	(O F)
4	(U O)	17	(U O)
5	(O F)	18	(U O)
6	(O O)	19	(O O)
7	(U O)	20	(O O)
8	(U O)	21	(O F)
9	(O F)	22	(O O)
10	(O O)	23	(U O)
11	(U F)	24	(U O)
12	(O O)	25	(U O)
13	(O O)	26	(O O)

SESSION 4B


SUB. 4
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y = 0
ΣU=8
STARVATION ABSORPTION BARRIER


SUB. 5
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y = 0
ΣU=8
STARVATION ABSORPTION BARRIER

SESSION 4B (CONT.)

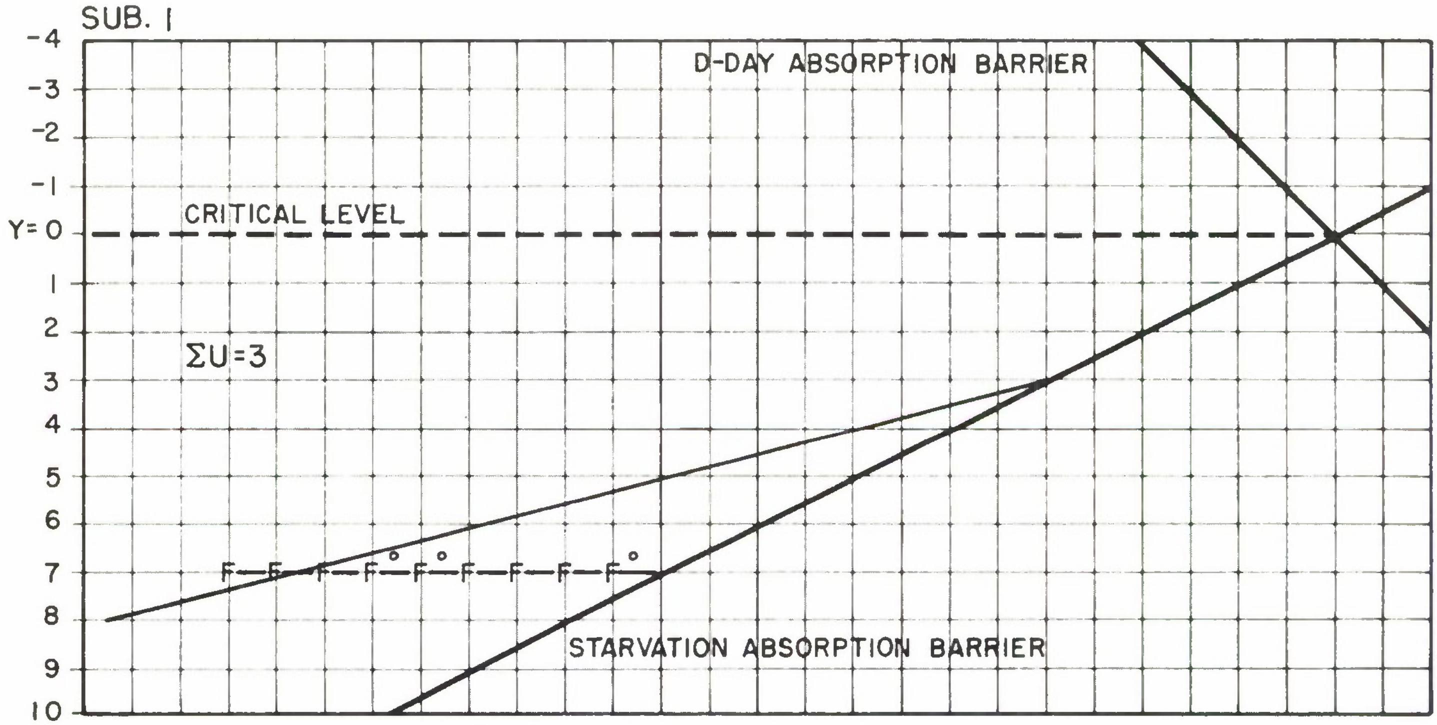


Session V A

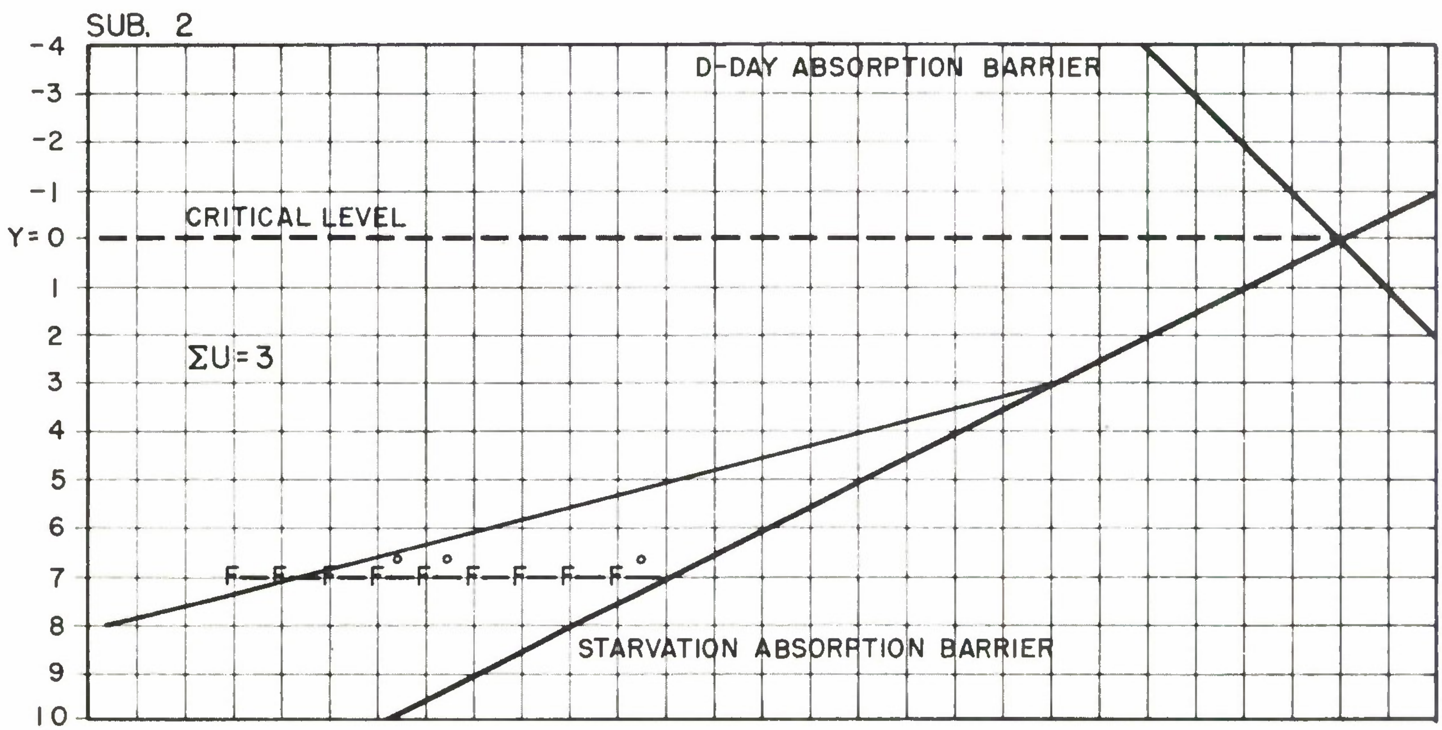
Values of parameters (u=.2 f=.2 a=3) : Starting conditions (x=9 y=7 L=30)

trial	outcome	trial	outcome
1	(O O)	6	(O O)
2	(O O)	7	(O O)
3	(O O)	8	(O O)
4	(U O)	9	(U O)
5	(U O)		

SESSION 5A

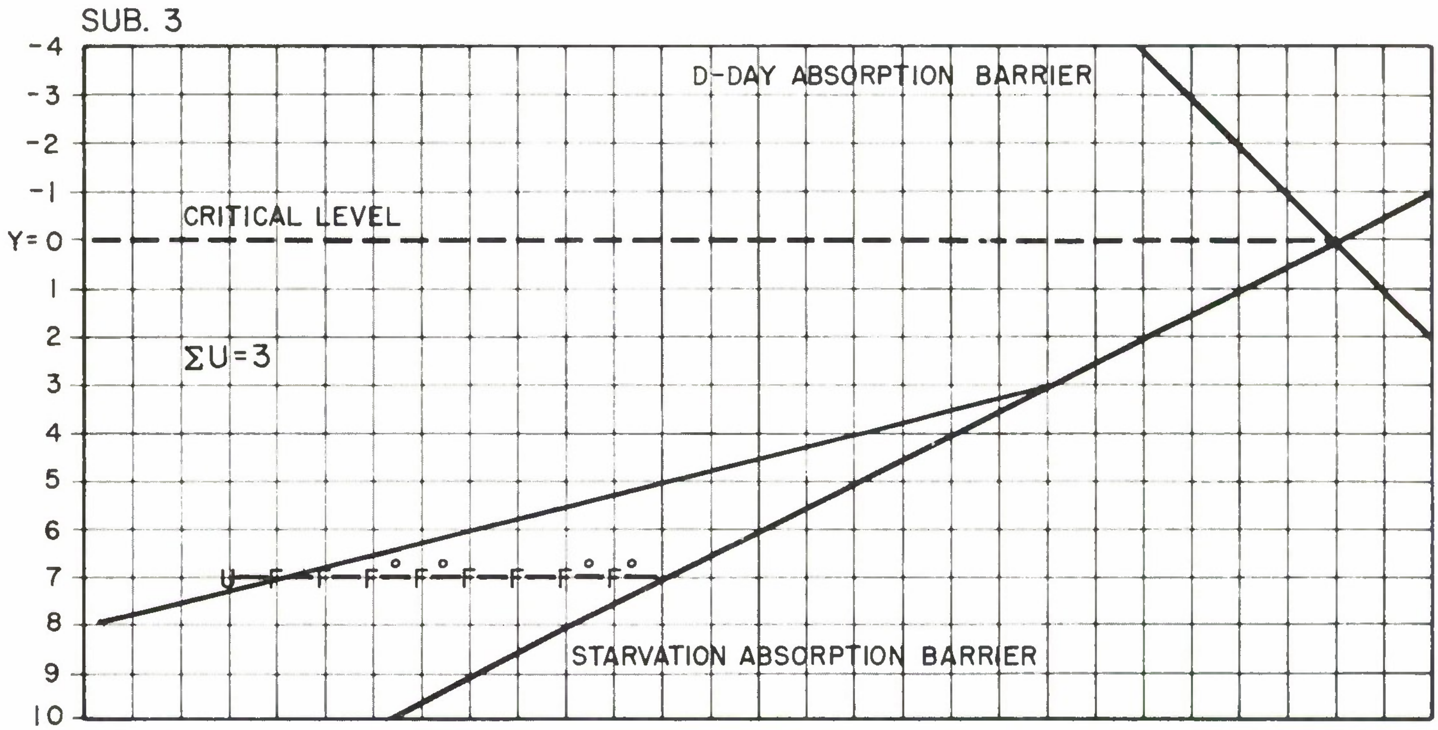


SUB. 1
-4
-3
-2
-1
Y= 0
1
2
3
4
5
6
7
8
9
10
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
ΣU=3
F—F—F—F—F—F—F—F—F
STARVATION ABSORPTION BARRIER



SUB. 2
-4
-3
-2
-1
Y= 0
1
2
3
4
5
6
7
8
9
10
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
ΣU=3
F—F—F—F—F—F—F—F—F
STARVATION ABSORPTION BARRIER

SESSION 5A (CONT.)



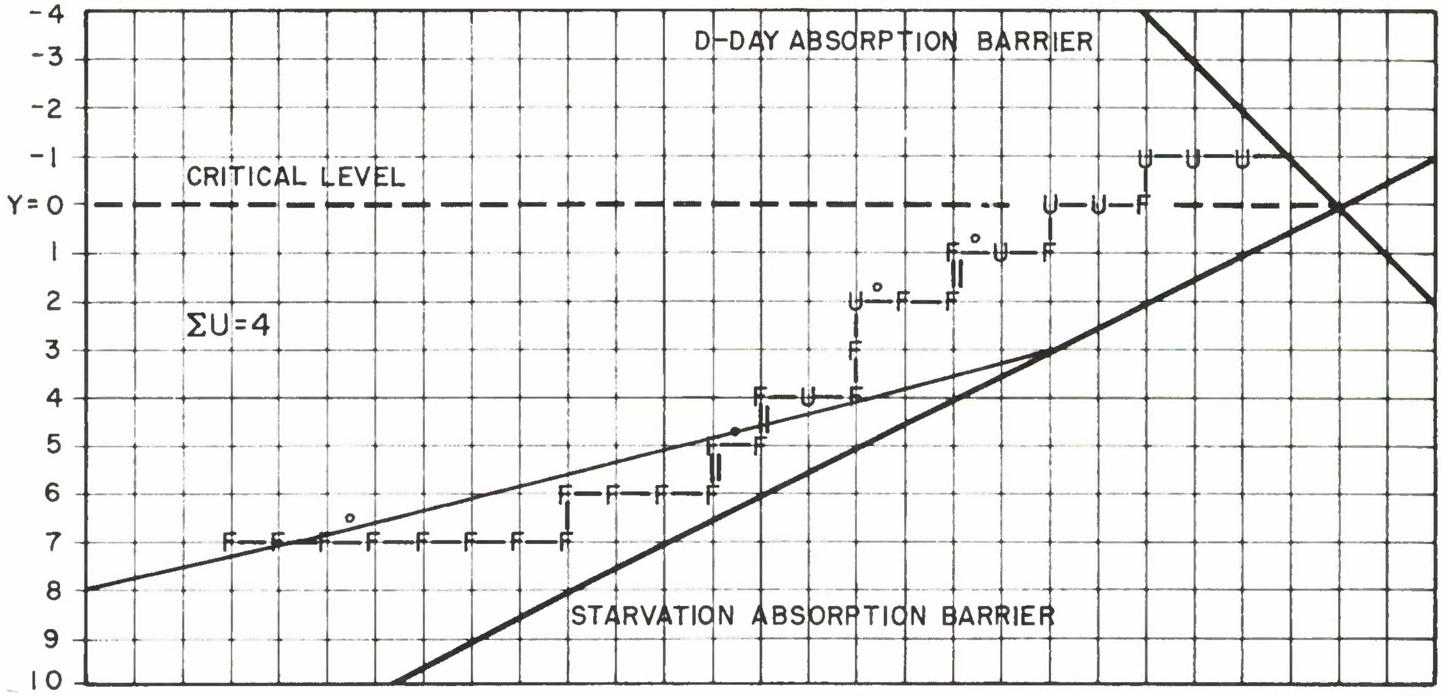
Session V B

Values of parameters (u=.2 f=.2 a=3) : Starting conditions (x=9 y=7 L=30)

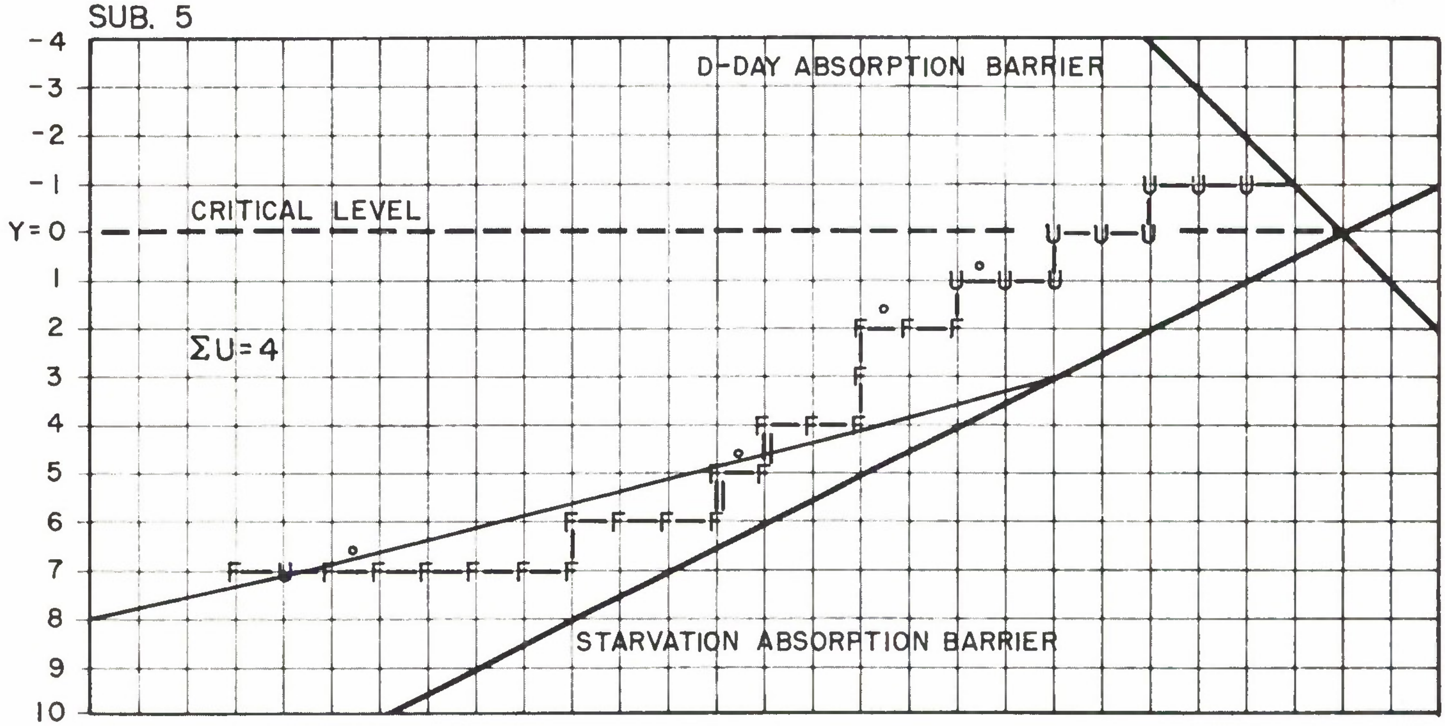
trial	outcome	trial	outcome
1	(O O)	16	(O O)
2	(O O)	17	(O F)
3	(U O)	18	(O F)
4	(O O)	19	(U O)
5	(O O)	20	(O O)
6	(O O)	21	(U F)
7	(O O)	22	(U O)
8	(O F)	23	(O O)
9	(O O)	24	(O F)
10	(O O)	25	(O O)
11	(O O)	26	(O O)
12	(U F)	27	(O F)
13	(U O)	28	(O O)
14	(U F)	29	(O O)
15	(O O)	30	(O O)

SESSION 5B

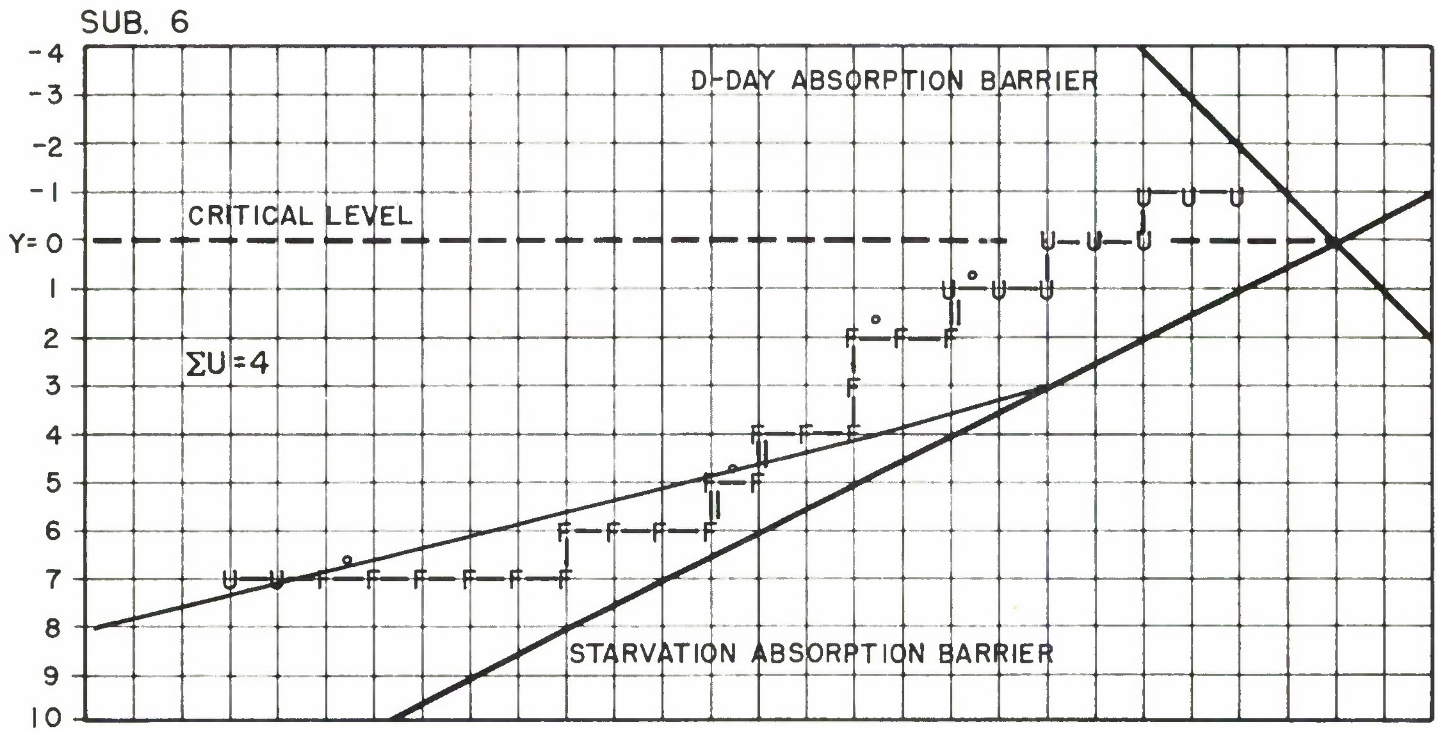
SUB. 4



SUB. 5



SESSION 5B (CONT.)

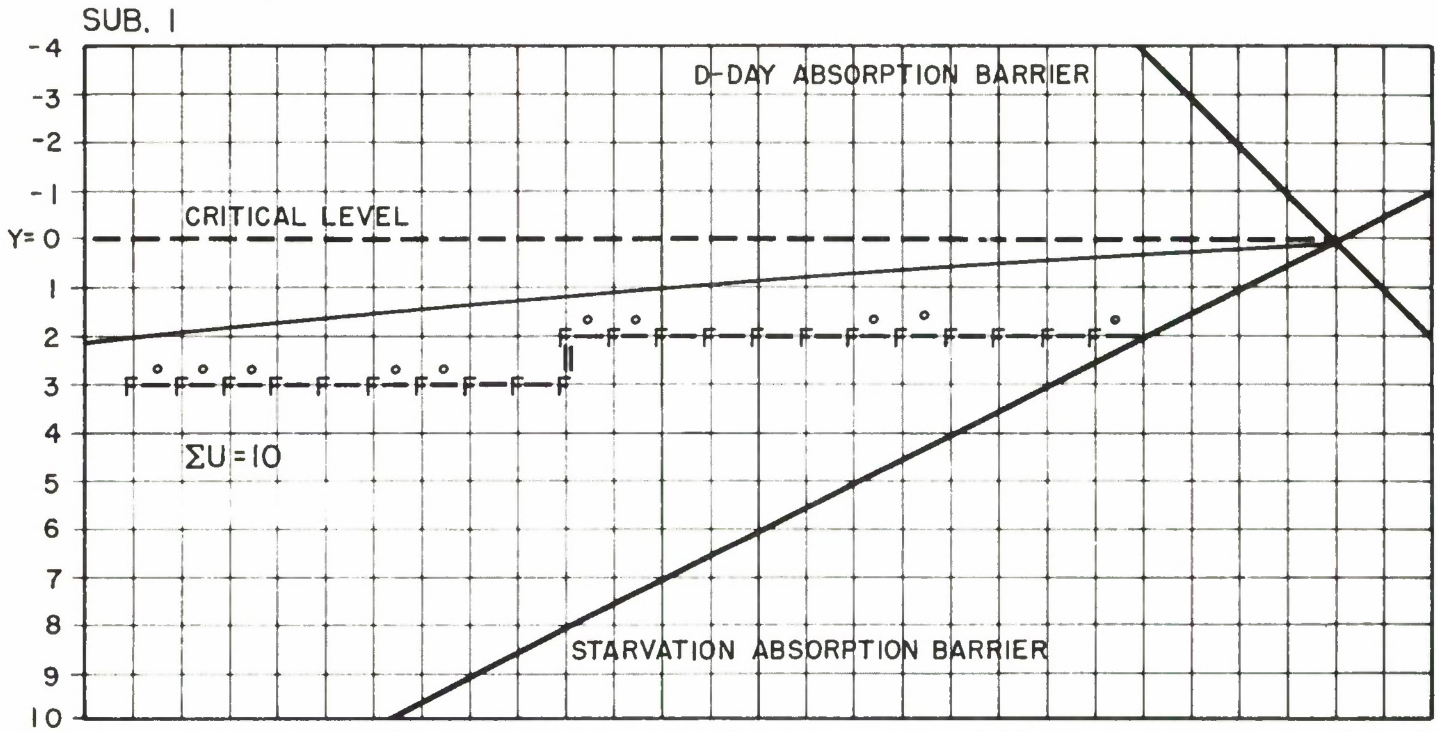


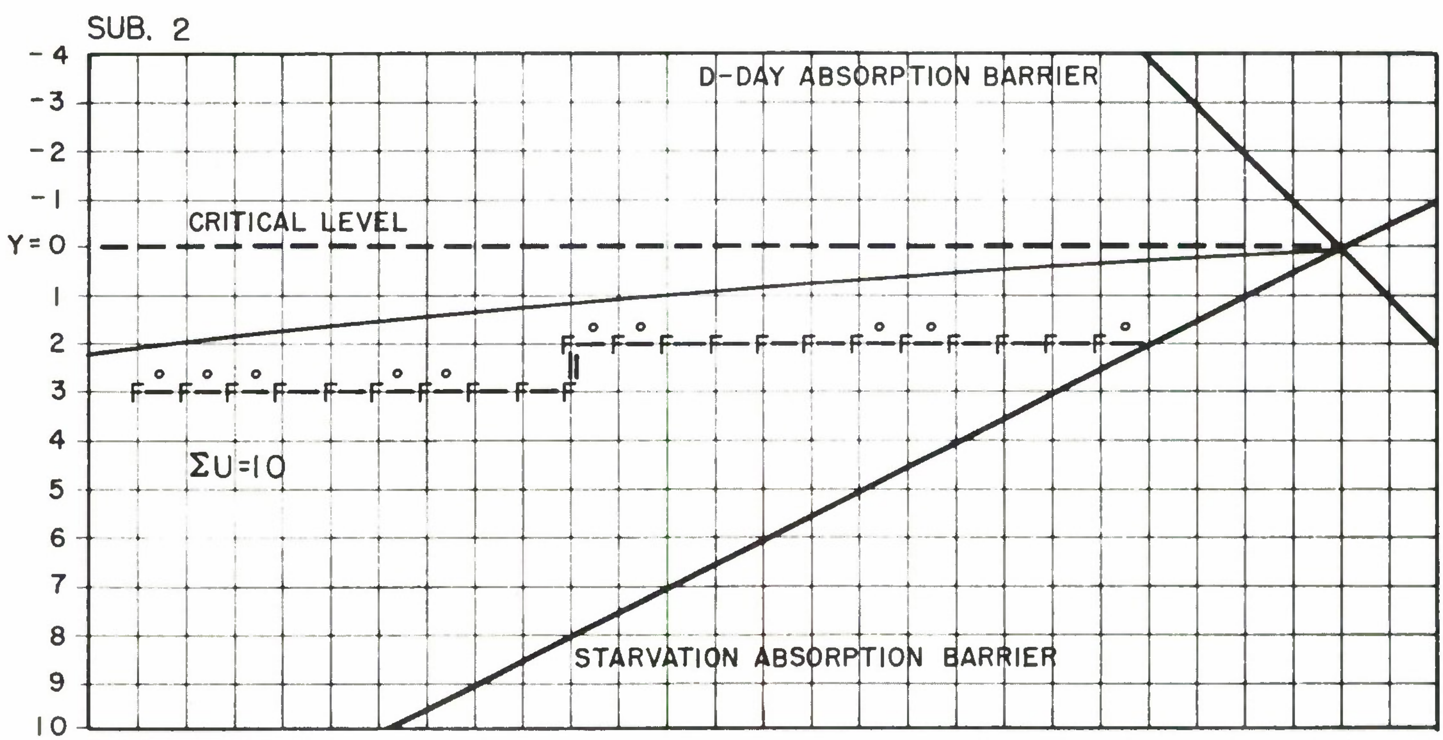
Session VI A

Values of parameters (u=.6 f=.1 a=3) : Starting conditions (x=19 y=3 L=28)

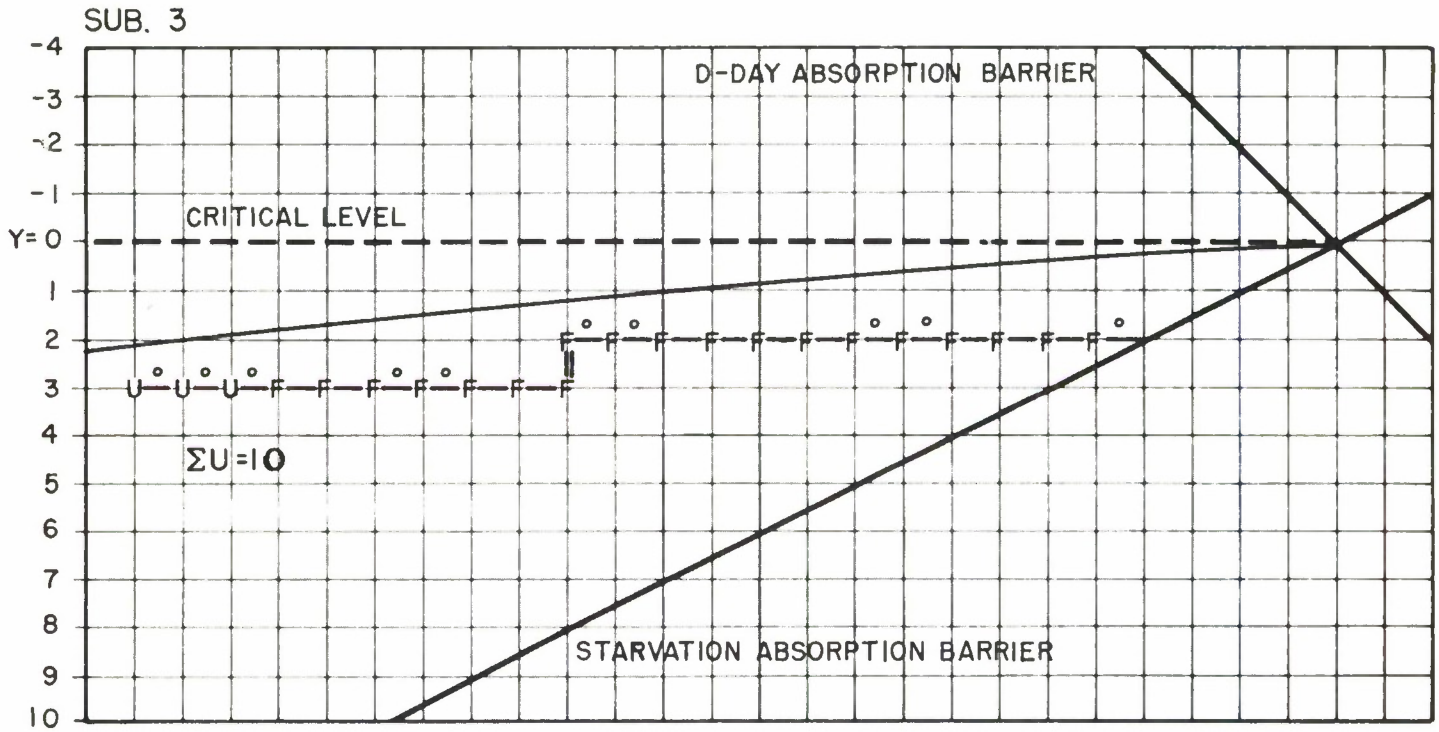
trial	outcome	trial	outcome
1	(U O)	12	(U O)
2	(U O)	13	(O O)
3	(U O)	14	(O O)
4	(O O)	15	(O O)
5	(O O)	16	(O O)
6	(U O)	17	(U O)
7	(U O)	18	(U O)
8	(O O)	19	(O O)
9	(O O)	20	(O O)
10	(U F)	21	(O O)
11	(U O)	22	(U O)

SESSION 6A


SUB. I
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y= 0
$\Sigma U=10$
STARVATION ABSORPTION BARRIER
-4
-3
-2
-1
1
2
3
4
5
6
7
8
9
10


SUB. 2
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y= 0
$\Sigma U=10$
STARVATION ABSORPTION BARRIER
-4
-3
-2
-1
1
2
3
4
5
6
7
8
9
10

SESSION 6A (CONT.)

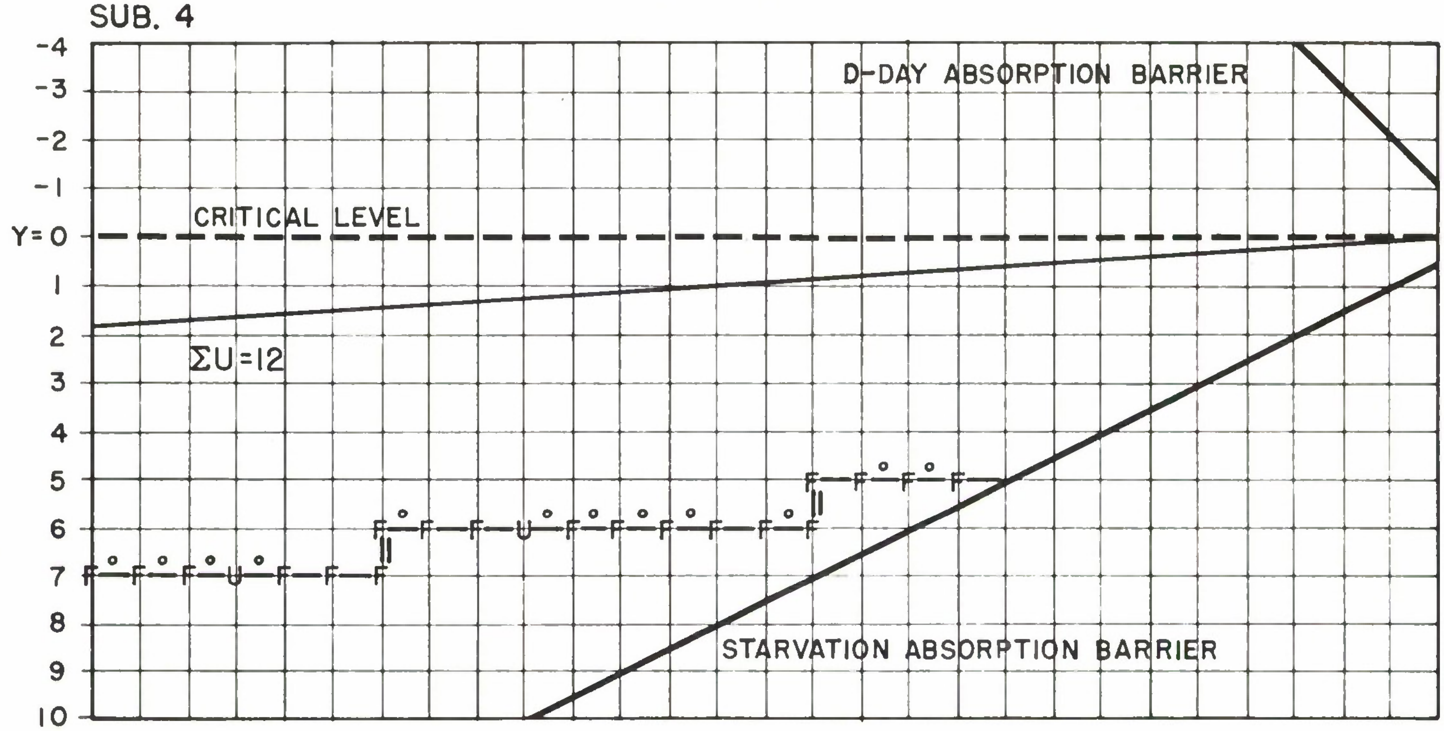


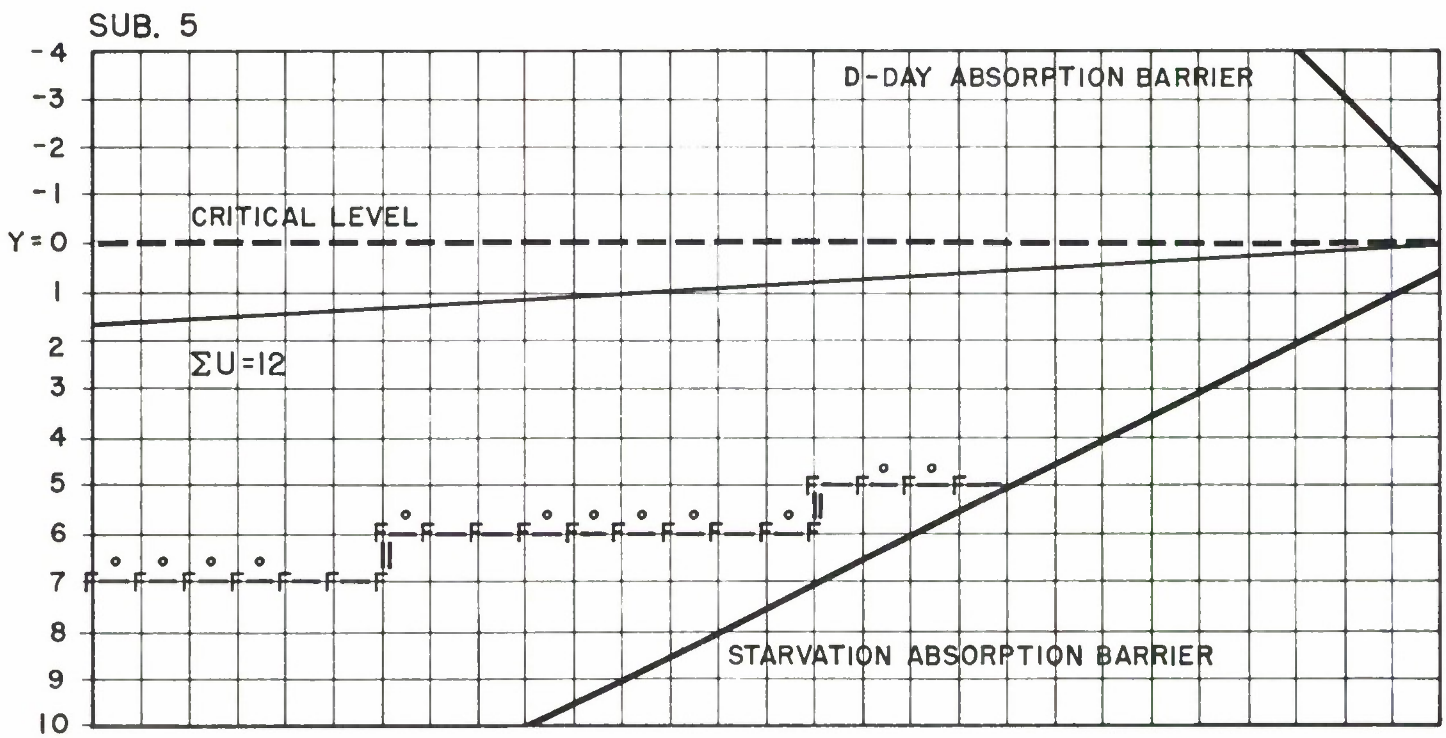
Session VI B

Values of parameters (u=.6 f=.1 a=3) : Starting conditions (x=15 y=7 L=36)

trial	outcome	trial	outcome
1	(U O)	11	(U O)
2	(U O)	12	(U O)
3	(U O)	13	(U O)
4	(U O)	14	(U O)
5	(O O)	15	(O O)
6	(O O)	16	(U O)
7	(U F)	17	(U F)
8	(U O)	18	(O O)
9	(O O)	19	(U O)
10	(O O)	20	(U O)
		21	(O O)

SESSION 6B


SUB. 4
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y=0
ΣU=12
STARVATION ABSORPTION BARRIER
-4
-3
-2
-1
1
2
3
4
5
6
7
8
9
10


SUB. 5
D-DAY ABSORPTION BARRIER
CRITICAL LEVEL
Y=0
ΣU=12
STARVATION ABSORPTION BARRIER
-4
-3
-2
-1
1
2
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6
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9
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9. Conclusion

We used a digital computer to solve the optimal decision problem of the F-E game, though G4 could not be regarded as a very complex game. We can anticipate from this point of view that it will be getting more and more difficult to get the optimal solution analytically in more elaborated F-E games . Therefore, our primary concern will be to get the computer program which will give us the optimal solution of each game.

As for the results of experiments, we have no right to discuss them in detail, because of the lack of sufficient data to uncover decision strategies actually employed by Ss. For that purpose we certainly need to repeat the same experiment over and over again with the same parameter values so that the locomotion traces of each subject cover a fairly large part of the IC diagram.

One thing we might be able to say is about the type of strategies used by most of the subjects on most of the games. It is the type of strategy one of the authors called the critical $\underline{x}$-value strategy or the economist's strategy[1]. Every subject seems to be trying to follow his own decision-shifting line . The critical $\underline{x}$-value seems to vary from subject to subject, and also vary with the values of parameters. At least, therefore, we may say that the observed strategies were not very far from optimal, contaminated, though, with individual's characteristic biases.

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1 Toda, op. cit.

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TODA, M. The design of a fungus-eater: A model of human behavior in an unsophisticated environment. Behavioral Science, 7, 164-183, 1962.

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TODA, M. Pre- and post- decisional processes of the "fungus-eater." Report No. 6, Division of Mathematical Psychology, Institute for Research, Pennsylvania. May 1963b.